Robust Mediation Analysis: The R Package robmed

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Abstract

Mediation analysis is one of the most widely used statistical techniques in the social, behavioral, and medical sciences. Mediation models allow to study how an independent variable affects a dependent variable indirectly through one or more intervening variables, which are called mediators. The analysis is often carried out via a series of linear regressions, in which case the indirect effects can be computed as products of coefficients from those regressions. Statistical significance of the indirect effects is typically assessed via a bootstrap test based on ordinary least-squares estimates. However, this test is sensitive to outliers or other deviations from normality assumptions, which poses a serious threat to empirical testing of theory about mediation mechanisms. The R package **robmed** implements a robust procedure for mediation analysis based on the fast-and-robust bootstrap methodology for robust regression estimators, which yields reliable results even when the data deviate from the usual normality assumptions. Various other procedures for mediation analysis are included in package **robmed** as well. Moreover, **robmed** introduces a new formula interface that allows to specify mediation models with a single formula, and provides various plots for diagnostics or visual representation of the results.

Keywords: mediation analysis, robust statistics, bootstrap, R.

This vignette is an updated version of Alfons, Ateş, and Groenen (2022b). Specifically, it has been adapted to reflect the change in default values for the bootstrap confidence intervals introduced in version 1.1.0.

1. Introduction

In the social, behavioral, and medical sciences, mediation analysis is a popular statistical technique for studying how an independent variable affects a dependent variable indirectly through an intervening variable called a mediator. For instance, Erreygers, Vandebosch, Vranjes, Baillien, and De Witte (2018) find that poor sleep quality in adolescents explains cyberbullying through anger, and Gaudiano, Herbert, and Hayes (2010) report that the believability of hallucinations after treatment for psychotic disorders mediates the relationship between the type of treatment and distress after treatment. Figure 1 shows a diagram of the mediation model in its simplest form, which is given by the equations

$$M = i_1 + aX + e_1, \tag{1}$$

$$Y = i_2 + bM + cX + e_2, (2)$$

$$Y = i_3 + c'X + e_3, (3)$$

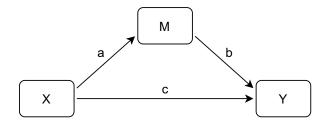


Figure 1: Diagram visualizing a simple mediation model.

where Y denotes the dependent variable, X the independent variable, M the hypothesized mediator, i_1 , i_2 , i_3 , a, b, c, and c' are regression coefficients to be estimated, and e_1 , e_2 , and e_3 are random error terms. The coefficients c and c' are called the direct effect and total effect, respectively, of X on Y. The product of coefficients ab is called the indirect effect of X on Y and constitutes the main parameter of interest in mediation analysis. Under the usual assumption of independent and normally distributed error terms e_1 , e_2 , and e_3 , it holds that c' = ab + c (e.g., MacKinnon, Warsi, and Dwyer 1995), and the same holds for the corresponding ordinary least-squares (OLS) estimates.

The indirect effect ab can be interpreted in the following way: a change of one unit in X explains a change of a units in M, which in turn explains a change of ab units in Y. It is therefore an important question whether or not to standardize the variables in some way. If the scales of the variables differ by orders of magnitude, certain coefficients may dominate the relationship c' = ab + c. However, variables used in mediation analysis often measure constructs that are aggregated from several rating-scale items (e.g., on a scale of 1–5). In such cases, a researcher may prefer not to standardize to keep the interpretation in terms of the original measurement scales. Similarly, a researcher may prefer not to standardize a binary X variable to keep the interpretation in terms of a change from one group to the other. For a more detailed discussion on whether or not to use standardized coefficients in mediation analysis, we refer to Hayes (2018, p. 519).

Mediation analysis goes back to Judd and Kenny (1981) and Baron and Kenny (1986), however their stepwise approach has been superseded by approaches that focus on the indirect effect. Sobel (1982) proposed a test for the indirect effect that assumes a normal distribution of the corresponding estimator, which is an unrealistic assumption for a product of coefficients. Bollen and Stine (1990), Shrout and Bolger (2002), MacKinnon, Lockwood, and Williams (2004), and Preacher and Hayes (2004, 2008) therefore advocate for a bootstrap test based on OLS regressions, which is the most frequently applied method for mediation analysis according to literature reviews of Wood, Goodman, Beckmann, and Cook (2008) and Alfons, Ates, and Groenen (2022a). More recently, several authors have emphasized that outliers or deviations from normality assumptions are detrimental to the reliability and validity of mediation analysis, and introduced more robust procedures. Zu and Yuan (2010) propose a bootstrap test after an initial data cleaning step, whereas Yuan and MacKinnon (2014) suggest a bootstrap test based on median regressions. Alfons et al. (2022a) combine the robust MM-estimator of regression (Yohai 1987) with the fast-and-robust bootstrap (Salibián-Barrera and Zamar 2002; Salibián-Barrera and Van Aelst 2008), and demonstrate that this procedure outperforms the aforementioned approaches for a wide range of error distributions (with different levels of skewness and kurtosis) and outlier configurations.

Various software packages are available for mediation analysis. The macro INDIRECT (Preacher and Hayes 2004, 2008) and its successor PROCESS (Hayes 2018) for SPSS (IBM Corp. 2021) and SAS (SAS Institute Inc. 2020) implement the bootstrap test based on OLS regressions. For the statistical computing environment R (R Core Team 2022), the general purpose packages psych (Revelle 2022) and MBESS (Kelley 2022) for statistical analysis in the behavioral sciences also provide functionality for a bootstrap test in mediation analysis. Package WRS2 (Mair and Wilcox 2020) is a collection of robust statistical methods, which offers mediation analysis via the bootstrap test after data cleaning proposed by Zu and Yuan (2010). Other packages concentrate on mediation analysis or specific aspects thereof. Package mediation (Tingley, Yamamoto, Hirose, Keele, and Imai 2014) is focused on causal mediation analysis in a potential outcome framework, and package medflex (Steen, Loeys, Moerkerke, and Vansteelandt 2017) implements recent developments in mediation analysis embedded within the counterfactual framework. Bayesian multilevel mediation models can be estimated with package bmlm (Vuorre 2021), while package mma (Yu and Li 2022) offers functionality for general multiple mediation analysis with continuous or binary/categorical variables. In addition, general purpose software for structural equation modeling such as Mplus (Muthén and Muthén 2017) or the R packages sem (Fox, Nie, and Byrnes 2022) and lavaan (Rosseel 2012) can be used for mediation analysis. The former also allows for maximum likelihood estimation with skew-normal, t, or skew-t error distributions (Asparouhov and Muthén 2016).

Despite the growing number of R packages that address mediation analysis, there are no common interfaces or class structures. Instead, each package uses its own way of specifying mediation models and storing the results. Additionally, only package **WRS2** contains some functionality for robust mediation analysis.

Package **robmed** (Alfons 2025) aims to address these issues. Its main functionality is the robust bootstrap procedure proposed in Alfons *et al.* (2022a), which is highly robust to outliers and other deviations from normality assumptions. Furthermore, **robmed** implements various other methods of estimating mediation models, as well as different tests for the indirect effects. All implemented methods share the same function interface and a clear class structure. In addition, **robmed** introduces a simple formula interface for specifying mediation models, and provides several plots for diagnostics or visualization of the results from mediation analysis.

Package **robmed** is available on CRAN (the Comprehensive R Archive Network, https://CRAN.R-project.org/) and can be installed from the R console with the following command:

R> install.packages("robmed")

The rest of the paper is organized as follows. Section 2 discusses various extensions of the simple mediation model, as well as the implemented methodology for estimation and testing. Implementation details are provided in Section 3, while Section 4 illustrates the use of package **robmed** with code examples. The final Section 5 concludes.

2. Methodology

We first provide overviews of the mediation models and estimation techniques supported by package **robmed** in Sections 2.1 and 2.2, respectively. Section 2.3 then gives technical details of the robust bootstrap procedure of Alfons *et al.* (2022a).

2.1. Extensions of the simple mediation model

The simple mediation model (1)–(3) can easily be extended in various ways, for instance with (i) multiple parallel mediators, (ii) multiple serial mediators, and (iii) multiple independent variables to be mediated. All those extensions may include additional control variables (covariates) as well.

Parallel multiple mediator model

In the parallel multiple mediator model, an independent variable X is hypothesized to influence a dependent variable Y through multiple mediators M_1, \ldots, M_k , while the mediator variables do not influence each other. A diagram of the model is displayed in Figure 2, and the corresponding regression equations are

$$M_j = i_j + a_j X + e_j, \qquad j = 1, \dots, k,$$
 (4)

$$Y = i_{k+1} + b_1 M_1 + \dots + b_k M_k + cX + e_{k+1}, \tag{5}$$

$$Y = i_{k+2} + c'X + e_{k+2}, (6)$$

where $i_1, \ldots, i_{k+2}, a_1, \ldots, a_k, b_1, \ldots, b_k, c$, and c' are regression coefficients to be estimated, and e_1, \ldots, e_{k+2} are random error terms. With the usual assumptions of independent and normally distributed error terms, we now have that $c' = \sum_{j=1}^k a_j b_j + c$. The main parameters of interest are the individual indirect effects a_1b_1, \ldots, a_kb_k , and it can also be of interest to make pairwise comparisons between the individual indirect effects or their absolute values (e.g., Hayes 2018, Chapter 5.1) if the hypothesized mediators are scaled similarly.

Serial multiple mediator model

A distinctive feature of the serial multiple mediator model is that the hypothesized mediators M_1, \ldots, M_k may influence each other in a sequential manner, unlike the parallel multiple mediator model in which the mediators do not affect one another. Figure 3 contains a diagram

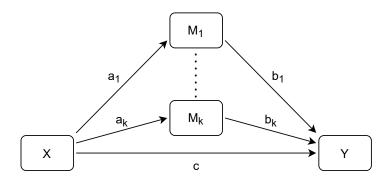


Figure 2: Diagram visualizing a parallel multiple mediator model.

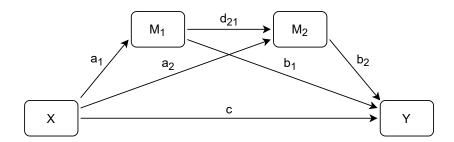


Figure 3: Diagram visualizing a serial multiple mediator model with two mediators.

of the model with two serial mediators, while the model in its general form is given by

$$M_{1} = i_{1} + a_{1}X + e_{1},$$

$$M_{2} = i_{2} + d_{21}M_{1} + a_{2}X + e_{2},$$

$$\vdots$$

$$(7)$$

$$M_k = i_k + d_{k1}M_1 + \dots + d_{k,k-1}M_{k-1} + a_kX + e_k,$$

$$Y = i_{k+1} + b_1M_1 + \dots + b_kM_k + cX + e_{k+1},$$
(8)

$$Y = i_{k+2} + c'X + e_{k+2}, (9)$$

where $i_1, \ldots, i_{k+2}, a_1, \ldots, a_k, d_{j1}, \ldots, d_{j,j-1}, j=2,\ldots,k, b_1,\ldots,b_k, c$, and c' are regression coefficients to be estimated, and e_1, \ldots, e_{k+2} are random error terms. It is easy to see that the serial multiple mediator model quickly grows in complexity with increasing number of mediators due to the combinatorial increase in indirect paths through the mediators (the number of indirect paths is given by $\sum_{j=1}^k \binom{k}{j}$ for k serial mediators). In package **robmed**, it is therefore only implemented for two and three mediators to maintain a focus on easily interpretable models. Here, we only discuss the model for two serial mediators, and we refer to Hayes (2018, p.169–171) for a diagram and a description of the various indirect effects in the case of three serial mediators.

For two serial mediators, the three indirect effects a_1b_1 $(X \to M_1 \to Y)$, a_2b_2 $(X \to M_2 \to Y)$, and $a_1d_{21}b_2$ $(X \to M_1 \to M_2 \to Y)$ are the main parameters of interest. However, not all pairwise comparisons of the indirect effects may be meaningful (even if the mediators are scaled similarly), as $a_1d_{21}b_2$ can be expected to be different in magnitude from a_1b_1 and a_2b_2 . Finally, we have that $c' = a_1b_1 + a_2b_2 + a_1d_{21}b_2 + c$ under the usual assumptions of independent and normally distributed error terms.

Multiple independent variables to be mediated

Instead of having multiple mediators, one can also allow for multiple independent variables X_1, \ldots, X_l to influence the dependent variable Y through a hypothesized mediator M. The resulting model is visualized in Figure 4 and defined by the equations

$$M = i_1 + a_1 X_1 + \dots + a_l X_l + e_1, \tag{10}$$

$$Y = i_2 + bM + c_1 X_1 + \dots + c_l X_l + e_2, \tag{11}$$

$$Y = i_3 + c_1' X_1 + \dots + c_l' X_l + e_3, \tag{12}$$

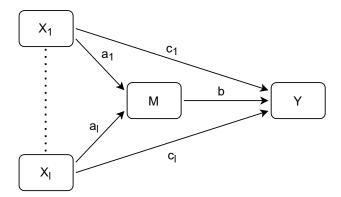


Figure 4: Diagram visualizing a mediation model with multiple independent variables.

where $i_1, i_2, i_3, a_1, \ldots, a_l, b, c_1, \ldots, c_l$, and c'_1, \ldots, c'_l are regression coefficients to be estimated, and e_1, e_2 , and e_3 are random error terms. The indirect effects a_1b, \ldots, a_lb are the main parameters of interest, and with the direct effects c_1, \ldots, c_l and total effects c'_1, \ldots, c'_l , it holds that $c'_j = a_jb + c_j$, $j = 1, \ldots, l$, under the usual independence and normality assumptions on the error terms. If the independent variables are on a comparable scale, it can also be of interest to make pairwise comparisons between the indirect effects or their absolute values.

This model is commonly used when the hypothesized mediator is the main (explanatory) variable of interest and its antecedents are being studied. Furthermore, an important special case of this model occurs when a categorical independent variable is represented by a group of dummy variables.

Control variables

In many study designs, it may be necessary to isolate the effects of the independent variables of interest from other factors. For instance, consider a study on whether exercise-induced feelings such as physical exhaustion mediate the relationship between physical activity and depression (cf. Pickett, Yardley, and Kendrick 2012). If the participants vary in demographics such as age and gender, the researcher may need to control for the effects of those variables (e.g., older people may be less capable of engaging in strenuous activities; Cerin, Leslie, Sugiyama, and Owen 2009). Such control variables should be added to all regression equations of a mediation model. This means that there is no intrinsic difference between independent variables of interest and control variables in terms of the model or its estimation. The difference is purely conceptual in nature: for the control variables, the estimates of the direct and indirect paths are not of particular interest to the researcher. Package **robmed** therefore allows to specify control variables separately from the independent variables of interest. Only for the latter, results for the indirect effects are included in the output.

While we omitted control variables from the above equations and diagrams for notational simplicity, package **robmed** supports additional control variables in all implemented models.

More complex models

The models described above do not exist in isolation and some of them can be combined. For instance, **robmed** supports parallel and serial multiple mediator models with multiple

independent variables of interest. Other variations of the mediation model, such as the moderated mediation and mediated moderation models (e.g., Muller, Judd, and Yzerbyt 2005) are out of scope for this paper. They are not yet implemented in package **robmed** but we aim to add support in future versions.

2.2. Overview of implemented methods

While package **robmed** is focused on the fast-and-robust bootstrap procedure for mediation analysis introduced by Alfons *et al.* (2022a), various other methods are available as well. Tables 1 and 2 provide an overview of the available methods together with the corresponding argument values to use in function test_mediation(), which implements mediation analysis in **robmed**.

A bootstrap test is considered state-of-the art for mediation analysis, with many authors advocating to use the bootstrap with ordinary least-squares (OLS) estimation of the coefficients in the mediation model (Bollen and Stine 1990; Shrout and Bolger 2002; MacKinnon et al. 2004; Preacher and Hayes 2004, 2008). However, a bootstrap test can easily be applied to other methods of estimation. For instance, the mediation model can also be estimated via regressions with more flexible error distributions such as the skew-normal, t, or skew-t distributions (see Azzalini and Arellano-Valle 2013, for maximum likelihood estimation of such regression models). Note that a similar procedure for mediation analysis, but using structural equation modeling, has been suggested in Asparouhov and Muthén (2016). Package robmed goes a step further in that it allows to select the best fitting error distribution via the Bayesian information criterion (BIC) (Schwarz 1978). In addition, other robust methods for mediation analysis are implemented in **robmed**. Yuan and MacKinnon (2014) proposed a bootstrap test that replaces OLS estimation with median regression. Zu and Yuan (2010) proposed to first winsorize the data via a Huber M-estimator of the covariance matrix, and then to perform a bootstrap test on the cleaned data with coefficient estimation based on the maximum likelihood covariance matrix. A discussion of advantages and disadvantages of those approaches, as well as a comparison in extensive simulation studies, can be found in Alfons et al. (2022a).

Besides bootstrap tests, variations of the Sobel test (Sobel 1982) are implemented in **robmed**. The Sobel test was originally proposed for maximum likelihood estimation of structural equation models, of which mediation models are a special case. It assumes a normal distribution of the indirect effect estimator and simplifies the calculation of the standard error by taking a first- or second-order approximation. This test has been criticized in the literature for its incorrect assumptions (e.g., MacKinnon, Lockwood, Hoffman, West, and Sheets 2002), and a bootstrap test is generally preferred. Nevertheless, the Sobel test can easily be generalized to other estimation methods, and it is implemented in **robmed** for all estimation procedures of the (simple) mediation model. We emphasize that the Sobel tests are implemented for comparisons in benchmarking experiments and that they are not recommended for empirical analyses.

2.3. Fast-and-robust bootstrap test for mediation analysis

The robust procedure of Alfons *et al.* (2022a) follows the state-of-the-art bootstrap approach for testing mediation (Bollen and Stine 1990; Shrout and Bolger 2002; MacKinnon *et al.* 2004; Preacher and Hayes 2004, 2008), but it replaces OLS regressions with the robust MM-

values to use in function test_mediation(). Table 1: Overview of bootstrap procedures for mediation analysis implemented in the R package robmed, and corresponding argument

"boot"	"boot"	"boot"	"boot"	"boot"	t est
"covariance"	"regression"	"regression"	"regression"	"regression"	method
TRUE FALSE	FALSE	FALSE	"median"	"MM" or TRUE	robust
	"select"	"gaussian"			family
Following multivariate winsorization, a bootstrap confidence interval is computed for coefficient estimation via the maximum likelihood estimator of the covariance matrix (Zu and Yuan 2010). A bootstrap confidence interval is computed for coefficient estimation via the maximum likelihood estimator of the covariance matrix.	Regression models with normal, skew-normal, t , or skew- t error distributions are estimated (Azzalini and Arellano-Valle 2013), and a bootstrap confidence interval is computed. The best fitting error distribution is selected via BIC.	A bootstrap confidence interval is computed based on OLS regressions (Bollen and Stine 1990; Shrout and Bolger 2002; MacKinnon <i>et al.</i> 2004; Preacher and Hayes 2004, 2008).	A bootstrap confidence interval is computed based on median regressions (Yuan and MacKinnon 2014).	MM-regression (Yohai 1987) and the fast-and-robust bootstrap (Salibián-Barrera and Zamar 2002) are used to construct a confidence interval (Alfons <i>et al.</i> 2022a).	Description
< <	<	<	<	<	Simple mediation
	<	<	<	<	Parallel mediators
	<	<	<	<	Serial mediators
	<	<	<	<	Multiple independent variables of interest
	<	<	<	<	Control variables

Control variables	>	>	>	>		
Multiple independent variables of interest						
Serial mediators						
erotsibəm ləllaraq						
noitsibəm əlqmiZ	>	>	>	>	>	>
Description	A variation of the Sobel test based on coefficient estimation via MM-regressions.	A variation of the Sobel test based on coefficient estimation via median regressions.	A variation of the Sobel test based on coefficient estimation via OLS regressions.	A variation of the Sobel test in which regression models with normal, skew-normal, t , or skew- t error distributions are estimated, with the best fitting distribution being selected via BIC.	Following multivariate winsorization, a variation of the Sobel test is applied based on coefficient estimation via the maximum likelihood estimator of the covariance matrix.	A variation of the Sobel test based on coefficient estimation via the maximum likelihood estimator of the covariance matrix.
family			"gaussian"	"select"		
robust	"MM" or TRUE	"median"	FALSE	FALSE	TRUE	FALSE
method	"regression"	"regression"	"regression"	"regression"	"covariance"	"covariance"
t est t	"sobel"	"sobel"	"sobel"	"sobel"	"sobel"	"sobel"

Table 2: Overview of variations of the Sobel test (Sobel 1982) for mediation analysis implemented in the R package robmed, and corresponding argument values to use in function test_mediation(). Those tests are included for benchmarking purposes and are not recommended for empirical analyses.

estimator of regression (Yohai 1987) and the standard bootstrap with the fast-and-robust bootstrap (Salibián-Barrera and Zamar 2002; Salibián-Barrera and Van Aelst 2008).

Robust regression

For a response variable Y, a (p+1)-dimensional random vector \mathbf{X} in which the first component is fixed at 1, and a random error term $\varepsilon \sim N(0, \sigma^2)$, the linear regression model is given by

$$Y = \mathbf{X}^{\top} \boldsymbol{\beta} + \varepsilon.$$

Denoting the corresponding observations by $(y_i, \mathbf{x}_i^{\top})^{\top}$, i = 1, ..., n, the MM-estimate of regression with loss function ρ (Yohai 1987) is defined as

$$\hat{\boldsymbol{\beta}}_n = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^n \rho\left(\frac{r_i(\boldsymbol{\beta})}{\hat{\sigma}_n}\right),\tag{13}$$

where $r_i(\beta) = y_i - \mathbf{x}_i^{\top} \beta$, i = 1, ..., n, are the residuals, and $\hat{\sigma}_n$ is an initial estimate of the residual scale. We take $\hat{\sigma}_n$ from a highly robust but inefficient S-estimator of regression (Rousseeuw and Yohai 1984; Salibián-Barrera and Yohai 2006), i.e.,

$$\hat{\sigma}_n = \min_{\beta} \hat{\sigma}_n(\beta),$$

where $\hat{\sigma}_n(\beta)$ is defined implicitly as the solution of

$$\frac{1}{n} \sum_{i=1}^{n} \rho_{S} \left(\frac{r_{i}(\boldsymbol{\beta})}{\hat{\sigma}_{n}(\boldsymbol{\beta})} \right) = \delta$$

with loss function ρ_S and $\delta = E\left[\rho_S\left(\frac{X}{\sigma}\right)\right]$ for a random variable $X \sim N(0, \sigma^2)$. For both ρ and ρ_S , we use Tukey's bisquare loss function defined as

$$\rho(x) = \begin{cases} \frac{x^6}{6c^4} - \frac{x^4}{2c^2} + \frac{x^2}{2}, & \text{if } |x| \le c\\ \frac{c^2}{6}, & \text{if } |x| > c. \end{cases}$$
 (14)

The value of the tuning constant c in ρ_S determines the robustness of the MM-estimator, and the value of c in ρ determines the efficiency (cf. Yohai 1987). By default, we set c=1.54764 in ρ_S for maximal robustness and c=3.443689 in ρ for 85% efficiency at the model with normally distributed errors.

Taking the derivative of the objective function in (13) and equating the derivative to $\mathbf{0}$ yields the system of equations

$$\sum_{i=1}^{n} \rho' \left(\frac{r_i(\boldsymbol{\beta})}{\hat{\sigma}_n} \right) \mathbf{x}_i = \mathbf{0}. \tag{15}$$

With weights

$$w_i = \frac{\rho'(r_i(\boldsymbol{\beta})/\hat{\sigma}_n)}{r_i(\boldsymbol{\beta})/\hat{\sigma}_n}, \qquad i = 1, \dots, n,$$

the system of equations in (15) can be rewritten as a weighted version of the normal equations:

$$\sum_{i=1}^{n} w_i r_i(\boldsymbol{\beta}) \mathbf{x}_i = \mathbf{0}. \tag{16}$$

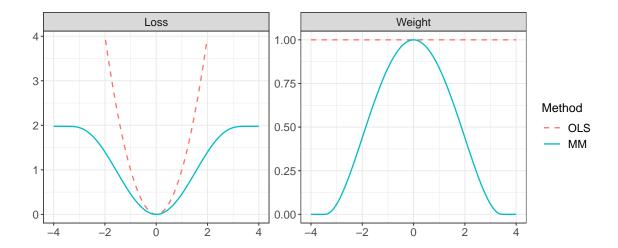


Figure 5: Loss function (left) and corresponding weight function (right) for OLS regression and the robust MM-estimator of regression.

Therefore, the MM-estimator can be seen as a weighted least-squares estimator with data-dependent weights. Those weights indicate how much each observation deviates, as an observation with a large residual (large deviation) receives a weight of 0 or close to 0, while an observation with a small residual (small deviation) receives a weight close to 1. The loss function from (14) and the resulting weight function are displayed in Figure 5, which also includes the loss function and weight function from OLS regression for comparison.

Fast-and-robust bootstrap

Here we briefly present the main idea of the fast-and-robust bootstrap. For a detailed discussion and complete derivations, the reader is referred to Salibián-Barrera and Zamar (2002) and Salibián-Barrera and Van Aelst (2008). There are two concerns with bootstrapping robust estimators:

- 1. Outliers may be oversampled in some bootstrap samples to the extent that those samples contain more outliers than the robust estimator can handle, in which case bootstrap confidence intervals become unreliable.
- 2. Robust estimators typically have higher computational complexity than their nonrobust counterparts, which is amplified when computing many bootstrap replicates.

In many empirical applications of mediation analysis, the first concern is unlikely to be an issue when using the MM-estimator of regression due to its high robustness. We therefore use the fast-and-robust bootstrap mainly for its computational efficiency, although the extra robustness does provide additional peace of mind.

To derive the fast-and-robust bootstrap for the MM-estimator, note that the solution of (16) can be written as

$$\hat{\boldsymbol{\beta}}_n = \left(\sum_{i=1}^n w_i \mathbf{x}_i \mathbf{x}_i^{\top}\right)^{-1} \sum_{i=1}^n w_i \mathbf{x}_i y_i.$$

For a bootstrap sample $(y_i^*, \mathbf{x}_i^{*\top})^{\top}$, i = 1, ..., n, one can compute $r_i^* = y_i^* - \mathbf{x}_i^{*\top} \hat{\boldsymbol{\beta}}_n$ and $w_i^* = \rho'(r_i^*/\hat{\sigma}_n)/(r_i^*/\hat{\sigma}_n)$ for i = 1, ..., n. It is important to note that $\hat{\boldsymbol{\beta}}_n$ and $\hat{\sigma}_n$ are computed in advance on the original sample such that the robustness weights w_i^* are inherited from the respective observations in the original sample. Then only a weighted least-squares fit is computed on the bootstrap sample to obtain

$$\hat{\boldsymbol{\beta}}_n^{\text{WLS}} = \left(\sum_{i=1}^n w_i^* \mathbf{x}_i^* \mathbf{x}_i^{*\top}\right)^{-1} \sum_{i=1}^n w_i^* \mathbf{x}_i^* y_i^*. \tag{17}$$

However, using (17) for the bootstrap distribution would not capture all the variability in the MM-estimator, as the robustness weights are not recomputed on the bootstrap samples. Nevertheless, a linear correction of the coefficients can be applied to overcome this loss of variability. The correction matrix only needs to be computed once based on the original sample and is given by

$$\boldsymbol{K}_n = \left(\sum_{i=1}^n \rho''(r_i/\hat{\sigma}_n)\mathbf{x}_i\mathbf{x}_i^{\top}\right)^{-1} \sum_{i=1}^n w_i\mathbf{x}_i\mathbf{x}_i^{\top}.$$

Then the fast-and-robust bootstrap replicate on the given bootstrap sample is computed as

$$\hat{\boldsymbol{\beta}}_{n}^{*} = \hat{\boldsymbol{\beta}}_{n} + \boldsymbol{K}_{n} \left(\hat{\boldsymbol{\beta}}_{n}^{\text{WLS}} - \hat{\boldsymbol{\beta}}_{n} \right). \tag{18}$$

Since the MM-estimator $\hat{\boldsymbol{\beta}}_n$ is consistent for $\boldsymbol{\beta}$ (Yohai 1987), $\sqrt{n}(\hat{\boldsymbol{\beta}}_n^* - \hat{\boldsymbol{\beta}}_n)$ has the same asymptotic distribution as $\sqrt{n}(\hat{\boldsymbol{\beta}}_n - \boldsymbol{\beta})$ (Salibián-Barrera and Van Aelst 2008; Salibián-Barrera and Zamar 2002).

Bootstrapping the indirect effects in mediation analysis

For simplicity, we focus on the indirect effect in the simple mediation model from (1)–(3). Similar calculations apply to the indirect effects in the mediation models described in Section 2.1. On each bootstrap sample, (18) is used to obtain estimates \hat{a} , \hat{b} , and \hat{c} of the coefficients in (1) and (2), and therefore estimates $\hat{a}\hat{b}$ of the indirect effect. Note that we do not perform the regression corresponding to (3) and instead estimate the total effect by $\hat{c}' = \hat{a}\hat{b} + \hat{c}$. With the empirical distribution of the indirect effect over the bootstrap samples, we construct a percentile-based confidence interval. By default, we report a simple percentile confidence interval (Davison and Hinkley 1997). Furthermore, we advocate to report the mean over the bootstrap distribution as the final point estimates of the indirect effect.

3. Package contents and implementation

We describe the included data set in Section 3.1, introduce the formula interface for specifying mediation models in Section 3.2, and briefly discuss the main functions as well as the class structure of package **robmed** in Section 3.3. Moreover, we load the package and the data in order to use them in code examples.

R> library("robmed")
R> data("BSG2014")

3.1. Example data

The BSG2014 data included in package **robmed** come from a business simulation game that was played by senior business administration students as part of a course at a Western European university. The simulation game was played twice, and a survey was conducted in three waves (before the first game, in between the two games, and after the second game). A total of 354 students formed 92 randomly assigned teams, and the responses of the individual students were aggregated to the team level. Leaving out teams with less than 50 percent response rate yields a sample size of n = 89 teams.

Below, we provide an overview of the variables that are used later on in the case studies in Section 4. For a complete description of the data, we refer to its R help file, which can be accessed from the console with ?BSG2014.

- ValueDiversity: Using the short Schwartz's value survey (Lindeman and Verkasalo 2005), the team members rated ten items on the importance of certain values (1 = not important, 10 = highly important). For each value item, the standard deviation of the individual responses across team members was computed, and the resulting standard deviations were averaged across the value items.
- TaskConflict: Using the intra-group conflict scale of Jehn (1995), the team members rated four items on the presence of conflict regarding the work on a 5-point scale (1 = none, 5 = a lot). The individual responses were aggregated by taking the average across items and team members.
- TeamCommitment: The team members indicated the extent to which they agree or disagree with four items on commitment to the team, which are based on Mowday, Steers, and Porter (1979), using a 5-point scale (1 = strongly disagree, 5 = strongly agree). The individual responses were aggregated by taking the average across items and team members.
- TeamScore: The team performance scores on the second game were computed at the end of the simulation through a mix of five objective performance measures: return on equity, earnings-per-share, stock price, credit rating, and image rating. The computation of the scores is handled by the simulation game software, and details can be found in Mathieu and Rapp (2009).
- SharedLeadership: Following Carson, Tesluk, and Marrone (2007), every team member assessed each of their peers on the question of "To what degree does your team rely on this individual for leadership?" using a 5-point scale (1 = not at all, 5 = to a very large extent). The leadership ratings were aggregated by taking the sum and dividing it by the number of pairwise relationships among team members.
- AgeDiversity: Following Harrison and Klein (2007), age diversity was operationalized by the standard deviation of the team members' ages.
- GenderDiversity: Gender diversity was measured with Blau's index, $1 \sum_j p_j^2$, where p_j is the proportion of team members in the j-th category (Blau 1977).
- Procedural Justice: Based on the intra-unit procedural justice climate scale of Li and Cropanzano (2009), the team members indicated the extent to which they agree or

disagree with four items on a 5-point scale (1 = strongly disagree, 5 = strongly agree). The individual responses were aggregated by taking the average across items and team members.

Interactional Justice: Using the intra-unit interactional justice climate scale of Li and Cropanzano (2009), the team members indicated the extent to which they agree or disagree with four items on a 5-point scale (1 = strongly disagree, 5 = strongly agree). The individual responses were aggregated by taking the average across items and team members.

TeamPerformance: Following Hackman (1986), the team members indicated the extent to which they agree or disagree with four items on the team's functioning, using a 5-point scale (1 = strongly disagree, 5 = strongly agree). The individual responses were aggregated by taking the average across items and team members.

To gain some insight into the distribution of those variables (including their ranges), we extract them from the data set and produce a summary:

```
ValueDiversity
                  TaskConflict
                                 TeamCommitment
                                                     TeamScore
                                                          : 49.00
Min.
       :1.105
                        :1.125
                                 Min.
                                         :3.000
                 Min.
                                                  Min.
1st Qu.:1.443
                 1st Qu.:1.500
                                 1st Qu.:3.750
                                                   1st Qu.: 90.00
Median :1.587
                Median :1.688
                                 Median :4.000
                                                  Median: 98.00
       :1.676
                        :1.761
                                         :3.992
                                                          : 95.72
Mean
                Mean
                                 Mean
                                                  Mean
                 3rd Qu.:2.000
3rd Qu.:1.916
                                  3rd Qu.:4.312
                                                   3rd Qu.:104.00
       :2.548
                        :2.938
                                         :5.000
                                                  Max.
                                                          :110.00
Max.
                 Max.
                                 Max.
SharedLeadership AgeDiversity
                                    GenderDiversity
                                                     ProceduralJustice
Min.
       :1.750
                         :0.0000
                                   Min.
                                           :0.0000
                                                             :3.375
                  Min.
                                                     Min.
1st Qu.:3.167
                  1st Qu.:0.5000
                                    1st Qu.:0.0000
                                                      1st Qu.:3.750
Median :3.333
                  Median : 0.8165
                                    Median :0.3750
                                                     Median :3.875
Mean
       :3.315
                  Mean
                         :0.9723
                                    Mean
                                           :0.3031
                                                     Mean
                                                             :3.908
3rd Qu.:3.583
                  3rd Qu.:1.2583
                                    3rd Qu.:0.3750
                                                      3rd Qu.:4.062
                                           :0.5000
Max.
       :4.667
                  Max.
                         :4.2720
                                   Max.
                                                     Max.
                                                             :4.500
Interactional Justice TeamPerformance
Min.
       :3.312
                      Min.
                             :3.000
1st Qu.:4.167
                      1st Qu.:3.667
Median :4.375
                      Median :4.000
Mean
       :4.379
                      Mean
                             :3.968
3rd Qu.:4.625
                      3rd Qu.:4.250
Max.
       :5.000
                      Max.
                             :4.750
```

For instance, the objective team performance scores in variable TeamScore range from 49 to 110.

3.2. Formula interface

The equations in the mediation model follow a specific structure regarding which variable is used as the response variable and which variables are the explanatory variables. Some R packages for mediation analysis, e.g., **mediation** (Tingley *et al.* 2014), require the user to specify one formula for each equation, which can be tedious and prone to mistakes, in particular for models with multiple mediators and multiple independent variables or control variables. Other packages, e.g., **psych** (Revelle 2022) or **MBESS** (Kelley 2022), do not provide a formula interface at all, despite formulas being the standard way of describing models in R.

For package **robmed**, we designed a formula interface that builds upon the standard formula interface in R, but allows to specify the mediation model with a single formula. As usual, the dependent variable is defined on the left hand side of the formula, and the independent variable is given on the right hand side. In addition, the functions m() and covariates() can be used on the right hand side to define the hypothesized mediators and any control variables, respectively. If multiple mediators are supplied, function m() provides the argument .model, which accepts the values "parallel" for parallel mediators (the default) and "serial" for serial mediators. The corresponding wrapper functions parallel_m() and serial_m() are available for convenience.

For example, a simple mediation model can be defined as follows (see also the case study in Section 4.1):

```
R> TeamCommitment ~ m(TaskConflict) + ValueDiversity
```

An example for a serial multiple mediator model is specified with the following formula (see also the case study in Section 4.2), where the serial mediators are listed in consecutive order from left to right:

```
R> TeamScore ~ serial_m(TaskConflict, TeamCommitment) + ValueDiversity
```

The formula specification for an example of a parallel multiple mediator model with control variables is given by (see also the case study in Section 4.3):

Note that different variables within m(), $parallel_m()$, $serial_m()$, and covariates() are separated by commas.

3.3. Main functions and class structure

The two main functions of package **robmed** are fit_mediation(), which implements various methods for the estimation of a mediation model, and test_mediation(), which performs statistical tests on the indirect effects in the mediation model. Furthermore, **robmed** follows a clear object-oriented design using S3 classes (Chambers 1992).

Function fit_mediation() is mainly intended to be used internally by test_mediation(), but it is also useful for a user who wants to compare different tests on the indirect effects for the same method of estimation, such that the estimation on the given sample only has to be performed once. It returns an object inheriting from class "fit_mediation". The currently

available subclasses are "reg_fit_mediation" if the mediation model was estimated via a series of regressions, and "cov_fit_mediation" if the model was estimated based on the covariance matrix of the involved variables.

We expect most users to find it more convenient to use test_mediation() directly in order to perform model estimation and testing the indirect effects with one function call. See Tables 1 and 2 for an overview of which argument values to use in test_mediation() for the various available methods. Furthermore, function robmed() is a wrapper function for the fast-and-robust bootstrap test of Alfons et al. (2022a). The results are returned as an object inheriting from class "test_mediation". The currently available subclasses are "boot_test_mediation" for bootstrap tests, and "sobel_test_mediation" for tests based on the normal approximation of Sobel (1982). Among other information, the component fit stores the estimation results as an object inheriting from class "fit_mediation". Objects of class "boot_test_mediation" also contain a component reps, which stores the bootstrap replicates as an object of class "boot", as returned by function boot() from package boot (Canty and Ripley 2021). It should be noted that the internal use of function boot() implies that the user can easily take advantage of parallel computing to reduce computation time.

Functions fit_mediation() and test_mediation() are implemented as generic functions. Two methods are available for both functions: one method uses the formula interface described in Section 3.2, while the default method provides an alternative way of specifying mediation models. Additionally, test_mediation() has a method for objects inheriting from class "fit_mediation", as returned by fit_mediation(). The default methods take the data set as their first argument in order to work nicely with the pipe operator, i.e., |> introduced in R version 4.1.0 or %>% from package magrittr (Bache and Wickham 2022). Arguments x, y, m, and covariates take character, integer, or logical vectors to select the independent variables, the dependent variable, the hypothesized mediators, and any additional control variables, respectively, from the data set. Note that this interface offers various ways to select the variables in a programmable manner. In case of multiple mediators, argument model allows to specify whether multiple mediators are treated as parallel or serial mediators.

Package **robmed** provides various accessor functions to extract relevant information from the returned objects, such as coef() and confint() methods. In addition, it contains the plot functions weight_plot() and ellipse_plot() for diagnostics, as well as ci_plot() to visualize confidence intervals and density_plot() to plot density estimates of the indirect effect estimators.

4. Illustrations: Using package robmed

We demonstrate the use of package **robmed** in three illustrative mediation analyses using the included data set BSG2014 (see Section 3.1). While the package and the data have already been loaded in Section 3, we store the seed to be used for the random number generator in an object, as it will be needed in all examples for the purpose of replicating the results.

R> seed <- 20150601

The following subsections provide examples for a simple mediation model (Section 4.1), a serial multiple mediator model (Section 4.2), as well as a parallel multiple mediator model with additional control variables (Section 4.3).

4.1. Simple mediation

In the first code example, we replicate parts of the empirical example of Alfons et al. (2022a). The illustrative hypothesis to be investigated is that task conflict mediates the relationship between team value diversity and team commitment. Using robmed's formula interface (see Section 3.2), we specify a simple mediation model with the dependent variable TeamCommitment on the left hand side. On the right hand side, we have the hypothesized mediator TaskConflict, which is wrapped in a call to function m(), as well as the independent variable ValueDiversity. As we will compare the robust bootstrap test of Alfons et al. (2022a) with the OLS bootstrap test (e.g., Preacher and Hayes 2004, 2008; Hayes 2018), we store the formula object for later use.

```
R> f_simple <- TeamCommitment ~ m(TaskConflict) + ValueDiversity
```

Next, we perform the two bootstrap tests using function test_mediation(). As usual for functions that fit models, we supply the model specification and the data via the formula and data arguments. By default, test_mediation() fits the mediation model via regressions (argument method = "regression") and performs a bootstrap test for the indirect effect (argument test = "boot") with 5000 bootstrap replications (argument R = 5000). In that case, setting robust = TRUE (the default) or robust = "MM" specifies the robust bootstrap procedure of Alfons et al. (2022a), while robust = FALSE yields the nonrobust OLS bootstrap test. Before each call to test_mediation(), we set the seed of the random number generator.

Other estimation methods for a bootstrap test can be specified via a combination of arguments, as outlined in Table 1.

Function test_mediation() returns an object inheriting from class "test_mediation". The corresponding summary() method shows the relevant information on the fitted models and emphasizes the significance tests of the total, direct, and indirect effects. For bootstrap tests, the information displayed by summary() by default stays within the bootstrap framework. For effects other than the indirect effect, asymptotic tests are performed using the normal approximation of the bootstrap distribution. That is, the sample mean and the sample standard deviation of the bootstrap replicates are used for asymptotic z tests. Furthermore, bootstrap estimates of all effects are shown in addition to the estimates on the original data. At the bottom of the output, the indirect effect is summarized by the estimate on the original data (column Data), the bootstrap estimate (i.e., the sample mean of the bootstrap replicates; column Boot), and the lower and upper limits of the confidence interval (columns Lower and Upper, respectively).

```
R> summary(robust_boot_simple)
```

Robust bootstrap test for indirect effect via regression

x = ValueDiversityy = TeamCommitment m = TaskConflict Sample size: 89 Outcome variable: TaskConflict Coefficients: Boot Std. Error z value Pr(>|z|) 0.1802 6.188 6.1e-10 *** 1.1182 1.1153 (Intercept) ValueDiversity 0.3197 0.3225 0.1079 2.988 0.00281 ** Robust residual standard error: 0.3033 on 87 degrees of freedom Robust R-squared: 0.1181, Adjusted robust R-squared: Robust F-statistic: 9.113 on 1 and Inf DF, p-value: 0.002539 Robustness weights: 4 observations are potential outliers with weight <= 1.3e-05: [1] 48 58 76 79 ___ Outcome variable: TeamCommitment Coefficients: Data Boot Std. Error z value Pr(>|z|) (Intercept) 4.54771 4.54820 0.29510 15.412 < 2e-16 *** TaskConflict -0.35750 -0.36137 0.13167 -2.744 0.00606 ** ValueDiversity 0.04233 0.04598 0.09897 0.465 0.64220 Robust residual standard error: 0.3842 on 86 degrees of freedom Robust R-squared: 0.1155, Adjusted robust R-squared: 0.09492 Robust F-statistic: 2.233 on 2 and Inf DF, p-value: 0.1072 Robustness weights: No potential outliers with weight < 0.0011 detected. The minimum weight is 0.15. Total effect of x on y: Boot Std. Error z value Pr(>|z|)ValueDiversity -0.07198 -0.07141 0.12080 -0.591 Direct effect of x on y: Data Boot Std. Error z value Pr(>|z|) ValueDiversity 0.04233 0.04598 0.09897 0.465 0.642 Indirect effect of x on y: Data Boot Lower Upper

```
TaskConflict -0.1143 -0.1174 -0.2633 -0.02329 ---
Level of confidence: 95 %

Number of bootstrap replicates: 5000
```

Number of bootstrap repricates. 5000

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

The above results are similar to those reported in Alfons et al. (2022a), but they are not identical due to a change in the random number generator in R version 3.6.0 and a change in the default type of bootstrap confidence intervals in **robmed** version 1.1.0. Specifically, the robust bootstrap test detects a significant indirect effect, as the confidence interval is strictly negative (albeit barely so at the 95% confidence level). This negative indirect effect is composed of a positive effect of value diversity on task conflict (see the output of the first regression model), and a negative effect of task conflict on team commitment (see the output of the second regression model). For further interpretation, recall that value diversity is measured as a standard deviation averaged over various value dimensions, and that task conflict and team commitment are measured as averages of items on a 5-point rating scale. On average, an increase in value diversity by one standard deviation explains an increase in task conflict by about 0.32 points, which in turn explains a decrease in team commitment by about 0.12 points. Furthermore, we observe that the direct effect of value diversity on team commitment is not significantly different from 0, meaning that value diversity affects team

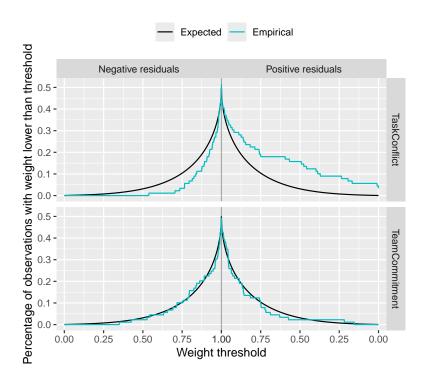


Figure 6: Diagnostic plot of the regression weights from the robust bootstrap procedure of Alfons *et al.* (2022a).

commitment only via the indirect path through task conflict. In the typology of mediations of Zhao, Lynch, and Chen (2010), we find indirect-only mediation.

Note that the output corresponding to the regression models is similar to that of the summary() method for "lmrob" objects from package robustbase (Maechler, Rousseeuw, Croux, Todorov, Ruckstuhl, Salibián-Barrera, Verbeke, Koller, Conceição, and Di Palma 2022), but it is shortened as the output is already rather long. In particular, we emphasize that the indices of potential outliers are displayed for each regression model. Those potential outliers should always be investigated further, as they may be interesting observations that could lead to new insights when studied separately (see Alfons et al. 2022a, for a detailed discussion on outliers in the mediation model).

Moreover, when the summary output for the robust bootstrap procedure of Alfons et al. (2022a) is printed, by default also a diagnostic plot is shown that allows to detect deviations from normality assumptions. Keep in mind that this procedure uses the robust MM-estimator of regression (Yohai 1987; Salibián-Barrera and Zamar 2002), which assigns robustness weights to all observations. Those weights can take any value in the interval [0, 1], with lower values indicating a higher degree of deviation. For a varying threshold on the horizontal axis, the diagnostic plot displays how many observations have a weight below this threshold. The plot is thereby split into separate panels for negative and positive residuals. For comparison, a reference line is drawn for the expected percentages under normally distributed errors.

Figure 6 shows the plot for this example. For the regression of the hypothesized mediator (TaskConflict) on the independent variable in the top row of the plot, it reveals much more downweighted observations with positive residuals than expected and fewer with negative residuals. This indicates right skewness with a heavy upper tail.

It is possible to suppress the plot by setting plot = FALSE in summary(). Then function weight_plot() can be used to create the diagnostic plot. In this example, Figure 6 can also be produced with the commands below.

```
R> weight_plot(robust_boot_simple) +
+ scale_color_manual("", values = c("black", "#00BFC4")) +
+ theme(legend.position = "top")
```

It should also be noted that the output from summary() is structured in a similar way as the output of the widely-used macro PROCESS (Hayes 2018), which implements the OLS bootstrap test for conditional process models such as the mediation model. The intention is to facilitate the use of package **robmed** for users of the PROCESS macro. While PROCESS constructs a bootstrap confidence interval for the indirect effect, it reports estimates on the original data and the usual normal-theory t tests for all other effects. In **robmed**, the same can be achieved by setting the argument type = "data" in the summary() method. The results from the regressions are then summarized in the usual way, as shown below for the OLS bootstrap.

```
R> summary(ols_boot_simple, type = "data")
Bootstrap test for indirect effect via regression
x = ValueDiversity
y = TeamCommitment
```

m = TaskConflict Sample size: 89 Outcome variable: TaskConflict Coefficients: Estimate Std. Error t value Pr(>|t|) (Intercept) 1.5007 0.2069 7.253 1.59e-10 *** ValueDiversity 0.1552 0.1209 1.283 0.203 Residual standard error: 0.3908 on 87 degrees of freedom Multiple R-squared: 0.01857, Adjusted R-squared: 0.007289 F-statistic: 1.646 on 1 and 87 DF, p-value: 0.2029 Outcome variable: TeamCommitment Coefficients: Estimate Std. Error t value Pr(>|t|) 0.252881 17.896 < 2e-16 *** (Intercept) 4.525460 TaskConflict -0.312124 0.103441 -3.017 0.00335 ** ValueDiversity 0.009362 0.117792 0.079 0.93684 Residual standard error: 0.3771 on 86 degrees of freedom Multiple R-squared: 0.0968, Adjusted R-squared: 0.07579 F-statistic: 4.608 on 2 and 86 DF, p-value: 0.01255 Total effect of x on y: Estimate Std. Error t value Pr(>|t|) ValueDiversity -0.03907 0.12201 -0.32 0.75 Direct effect of x on y: Estimate Std. Error t value Pr(>|t|) ValueDiversity 0.009362 0.079 0.937 0.117792 Indirect effect of x on y: Data Boot Lower Upper TaskConflict -0.04843 -0.0496 -0.1579 0.02916 Level of confidence: 95 %

Unlike the robust bootstrap test above, the OLS bootstrap does not detect a significant indirect effect, since the confidence interval covers 0. Due to the influential heavy tail indicated

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Number of bootstrap replicates: 5000

by the diagnostic plot in Figure 6, the results of the robust bootstrap test can be considered more reliable.

Methods for common generic functions to extract information from objects are implemented in **robmed**, such as a **coef()** method to extract the relevant effects of the mediation model, and **confint()** to extract confidence intervals of those effects.

R> coef(robust_boot_simple)

```
a b Total Direct Indirect 0.32250035 -0.36136840 -0.07141343 0.04598409 -0.11739752
```

R> confint(robust_boot_simple)

```
2.5 % 97.5 % a 0.1109474 0.53405335 b -0.6194382 -0.10329860 Total -0.3081750 0.16534815 Direct -0.1479943 0.23996248 Indirect -0.2632805 -0.02329195
```

While the confidence intervals in this example do not add much in terms of interpretation over the output of summary(), the latter reports significance tests instead of confidence intervals for the effects other than the indirect effect. For researchers who prefer to report confidence intervals, the confint() method allows to easily extract this information.

For objects corresponding to bootstrap tests (class "boot_test_mediation"), argument type of the coef() method allows to specify whether to extract the bootstrap estimates ("boot", the default) or the estimates on the original data ("data"). Similarly, argument type of the confint() method allows to specify whether the confidence intervals for the effects other than the indirect effect should be bootstrap confidence intervals ("boot", the default) or normal theory intervals based on the original data ("data").

```
R> coef(ols_boot_simple, type = "data")
```

```
a b Total Direct Indirect 0.155177478 -0.312124268 -0.039072956 0.009361701 -0.048434657
```

R> confint(ols_boot_simple, type = "data")

```
2.5 % 97.5 % a -0.08521602 0.39557098 b -0.51775881 -0.10648972 Total -0.28157636 0.20343045 Direct -0.22480114 0.24352454 Indirect -0.15792757 0.02916059
```

In addition, argument parm allows to specify which coefficients or confidence intervals to extract.

While the bootstrap tests implemented in test_mediation() construct a confidence interval for the indirect effect based on a pre-specified confidence level $1 - \alpha$, function p_value() allows to analyze the bootstrap distribution and extract the smallest significance level α for which the $(1 - \alpha) \cdot 100\%$ confidence interval of the indirect effect does not contain 0.

```
R> p_value(robust_boot_simple, parm = "Indirect")
Indirect
    0.0117
R> p_value(ols_boot_simple, parm = "Indirect")
Indirect
    0.2246
```

It should be noted that depending on the seed of the random number generator, the p value for the robust bootstrap test may fall below or above the common 5% significance level. But given the arbitrariness of this threshold, we find that robust bootstrap test shows at least weak evidence against the null hypothesis of no mediation, whereas the OLS bootstrap fails to do so.

Several plots are implemented to visualize the results of mediation analysis. They can be applied to each object individually, but it is also possible to combine multiple objects from mediation analysis into a list in order to compare different methods graphically. If names are given to the list elements, those names will be used by the plots to identify the different methods. Note that we use the name "ROBMED" here to refer to the robust bootstrap test.

Function density_plot() plots the density estimates of the indirect effect. It also adds vertical lines for the point estimates and illustrates the confidence intervals by shaded areas. The plot resulting from the following command is displayed in Figure 7.

```
R> density_plot(boot_list)
```

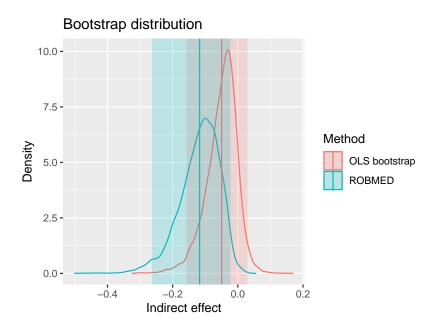


Figure 7: Density plot of the bootstrap distributions of the indirect effect, obtained via the OLS bootstrap and the robust bootstrap procedure of Alfons *et al.* (2022a). The vertical lines indicate the the respective point estimates of the indirect effect and the shaded areas represent the confidence intervals.

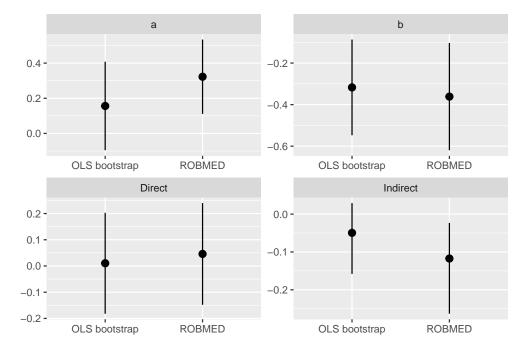


Figure 8: Point estimates and 95% confidence intervals for selected effects in the mediation model, estimated via the OLS bootstrap and the robust bootstrap procedure of Alfons $et\ al.$ (2022a).

In order to aid with interpretation of the results from mediation analysis, function ci_plot() allows to visualize the point estimates and confidence intervals of selected effects. The direct effect and the indirect effect are plotted by default, as the typology of mediations of Zhao et al. (2010) is based on those two effects. Nevertheless, argument parm can be used to specify which effects to plot. Figure 8 contains the plot created by the command below.

```
R> ci_plot(boot_list, parm = c("a", "b", "Direct", "Indirect"))
```

Finally, function ellipse_plot() produces a bivariate scatterplot together with a tolerance ellipse that illustrates how well the regression results represent the data, exploiting the relationship between regression coefficients and the covariance matrix. For the robust bootstrap procedure of Alfons et al. (2022a), the robustness weights from the robust regression estimator can be used to compute a weighted sample covariance matrix, from which the tolerance ellipse is computed. It is important to note that such a weighted covariance matrix is not Fisher consistent (that is, the functional form of the estimator applied to the model distribution does not equal the true covariance matrix), as observations are also expected to be downweighted when all observations follow the model. However, it is straightforward to obtain a correction for a Fisher consistent covariance matrix (see Appendix A).

For instance, we can produce such a plot with the independent variable on the horizontal axis and the hypothesized mediator on the vertical axis. In that case the plot represents the results from the regression of the hypothesized mediator on the independent variable. As the independent variable is the only explanatory variable in this regression model, the plot also adds lines representing the respective regression coefficients.

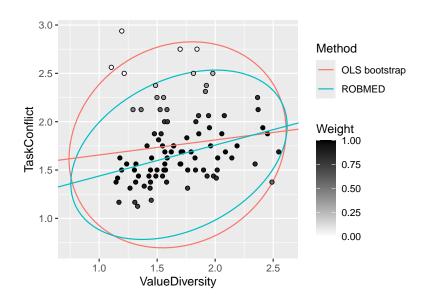


Figure 9: Diagnostic plot with tolerance ellipses for the OLS bootstrap and the robust bootstrap procedure of Alfons *et al.* (2022a).

The resulting plot is shown in Figure 9. Since we are comparing a nonrobust method with a robust method that assigns a robustness weight to each observation, by default those robustness weights are visualized by plotting the points on a grayscale. Clearly, the tolerance ellipse corresponding to the robust method fits the main bulk of the data better, as the tolerance ellipse corresponding to the nonrobust method contains more empty space. This plot further suggests that the detected potential outliers (see also the printed output of summary() above) are a result of the heavy upper tail in the hypothesized mediator (TaskConflict).

All plot functions in **robmed** allow customization via the underlying package **ggplot2** (Wickham 2016). Arguments can be passed down to <code>geom_xxx()</code> functions, and additional elements can be added to the plot as usual with the + operator. We refer to the R help files of the plots for some examples. Nevertheless, there are limits to the customization, in particular for the plots that contain various elements such as the diagnostic plot with the tolerance ellipse.

For further customization, **robmed** provides the workhorse functions <code>setup_weight_plot()</code>, <code>setup_ci_plot()</code>, <code>setup_density_plot()</code>, and <code>setup_ellipse_plot()</code>. They do not produce the plot, but they extract the relevant information to be displayed. They are useful for users who want to create similar plots, but who want more control over what information to display or how to display that information. With the commands below, we manually produce the same plot as before, but only plot the tolerance ellipses and the data without the regression lines. Figure 10 displays the resulting plot.

4.2. Serial multiple mediators

The second example extends the simple mediation model from the previous section to a serial multiple mediator model. We investigate the following illustrative hypothesis: value diversity negatively impacts team commitment through increased task conflict, and in turn task conflict negatively affects team performance through decreased team commitment. Here the dependent variable is an objective assessment of team performance, measured by the team's score on the simulation game.

The corresponding formula is stored in an object for later use and contains the dependent variable TeamScore on the left hand side. On the right hand side, the hypothesized serial mediators TaskConflict and TeamCommitment are wrapped in a call to serial_m(), separated by the usual + operator from the independent variable ValueDiversity.

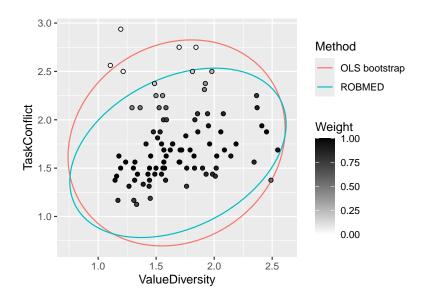


Figure 10: Customized diagnostic plot with tolerance ellipses but without regression lines for the OLS bootstrap and the robust bootstrap procedure of Alfons *et al.* (2022a).

Function test_mediation() is then called in the same way as in the previous section to compare the robust bootstrap procedure of Alfons *et al.* (2022a) with the nonrobust OLS bootstrap.

The output of summary() looks very similar to that of the previous example, except that the last part shows results for multiple indirect effects. In order to save space, we only print the objects returned by test_mediation(), which shows the results for the bootstrap confidence intervals of the indirect effects. For completeness, the full summary() output of the robust bootstrap test can be found in Appendix B.

```
R> robust_boot_serial
```

Robust bootstrap tests for indirect effects via regression

```
Indirect effects of x on y:

Data Boot Lower Upper
Total -0.2772 -0.2587 -3.430 2.6489
Indirect1 0.5216 0.5193 -1.130 2.3905
Indirect2 0.4698 0.5363 -1.594 3.0285
Indirect3 -1.2686 -1.3142 -3.140 -0.2413
```

```
Indirect effect paths:
 Label
           Path
 Indirect1 ValueDiversity -> TaskConflict
                                            -> TeamScore
 Indirect2 ValueDiversity -> TeamCommitment -> TeamScore
 Indirect3 ValueDiversity -> TaskConflict
                                            -> TeamCommitment -> TeamScore
Level of confidence: 95 %
Number of bootstrap replicates: 5000
R> ols_boot_serial
Bootstrap tests for indirect effects via regression
Indirect effects of x on y:
              Data
                       Boot Lower Upper
Total
          -0.21468 -0.31249 -2.998 2.119
Indirect1 0.18321 0.08061 -1.170 1.169
Indirect2 0.09533 0.11411 -1.894 2.310
Indirect3 -0.49322 -0.50721 -1.782 0.287
Indirect effect paths:
 Label
           Path
 Indirect1 ValueDiversity -> TaskConflict
                                            -> TeamScore
 Indirect2 ValueDiversity -> TeamCommitment -> TeamScore
 Indirect3 ValueDiversity -> TaskConflict
                                            -> TeamCommitment -> TeamScore
Level of confidence: 95 %
Number of bootstrap replicates: 5000
```

Note that the row labeled Total in the above output contains the results for the sum of the three individual indirect effects. We observe that the robust bootstrap test detects mediation in the indirect path that goes first through task conflict and subsequently through team commitment (effect Indirect3), whereas none of the indirect effects are significant in the OLS bootstrap.

We can use function weight_plot() to produce a diagnostic plot to investigate deviations from normality assumptions that could explain the differences in results for the two methods. The plot created with the commands below is shown in Figure 11.

```
R> weight_plot(robust_boot_serial) +
+ scale_color_manual("", values = c("black", "#00BFC4")) +
+ theme(legend.position = "top")
```

As in the previous example, we see that there are strong indications of a heavy upper tail in the regression of task conflict on value diversity (top row of Figure 11) and a heavy lower tail

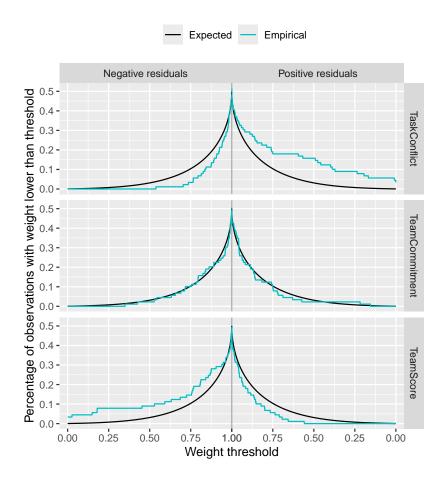


Figure 11: Diagnostic plot of the regression weights from the robust bootstrap procedure of Alfons *et al.* (2022a) in the example for a serial multiple mediator model.

in the regression for objective team performance (bottom row of Figure 11), which makes the results of the OLS bootstrap unreliable.

4.3. Parallel multiple mediators and control variables

In the final example, we investigate the illustrative hypothesis that procedural justice and interactional justice are parallel mediators for the relationship between shared leadership and the team's perception of its performance, controlled for diversity in age and gender within the team. Unlike in the previous section, the dependent variable is a subjective measure of team performance, as evaluated by the team members themselves.

In **robmed**'s formula interface, parallel multiple mediators can be specified by wrapping multiple variables in a call to function <code>parallel_m()</code>, and control variables can be specified in a similar manner with function <code>covariates()</code>.

R> f_parallel <-

- + TeamPerformance ~ parallel_m(ProceduralJustice, InteractionalJustice) +
- + SharedLeadership + covariates(AgeDiversity, GenderDiversity)

Function test_mediation() is then called in the usual way to compare the robust bootstrap test of Alfons *et al.* (2022a) and the nonrobust OLS bootstrap.

To save space, we again do not show the entire summary() of the resulting objects, but only the results for the indirect effect by printing the objects. Appendix B contains the full summary() output of the robust bootstrap test, including the diagnostic plot of the regression weights.

```
R> robust_boot_parallel
```

Robust bootstrap tests for indirect effects via regression

```
Indirect effects of x on y:
```

```
        Data
        Boot
        Lower
        Upper

        Total
        0.24177
        0.25038
        0.100035
        0.4430

        ProceduralJustice
        0.07967
        0.09043
        0.001816
        0.2386

        InteractionalJustice
        0.16210
        0.15995
        0.049338
        0.2885
```

Level of confidence: 95 %

Number of bootstrap replicates: 5000

```
R> ols_boot_parallel
```

Bootstrap tests for indirect effects via regression

Indirect effects of x on y:

```
DataBootLowerUpperTotal0.149030.153230.0300000.3069ProceduralJustice0.081850.092020.0095460.2328InteractionalJustice0.067180.06121-0.0080660.1452
```

Level of confidence: 95 %

Number of bootstrap replicates: 5000

As for the serial multiple mediator model, the row Total in the output above contains the results for the sum of the two individual indirect effects through the hypothesized mediators ProceduralJustice and InteractionalJustice. We observe that the robust procedure of Alfons et al. (2022a) finds support for mediation via both procedural justice and interactional

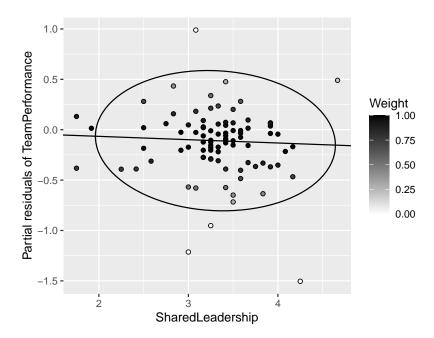


Figure 12: Diagnostic plot with a tolerance ellipse for partial residuals in a multiple parallel mediator model.

justice (since the confidence intervals are strictly positive), whereas only the indirect effect of procedural justice is significant at the 5% level for the OLS bootstrap.

The output of summary() in Appendix B, indicates that the residual distributions are fairly normal (see the diagnostic plot in Figure 13), but that there is a small number of potential outliers that should be investigated further. As the regression of the dependent variable (TeamPerformance) on the remaining variables shows more deviations from normality than the other regressions (bottom row of Figure 13), we use ellipse_plot() to create the diagnostic plot with a tolerance ellipse that is related to the regression results. If several explanatory variables are included, it is often more insightful to plot the partial residuals on the vertical axis, i.e., to subtract from the response the linear predictor without the variable that is displayed on the horizontal axis. This has the additional advantage that the regression coefficient can be visualized by a line, which is otherwise not possible in the case of multiple explanatory variables. With function ellipse_plot(), plotting the partial residuals can easily be achieved by setting the argument partial = TRUE. With the command below, we plot the partial residuals of team performance against the independent variable shared leadership. Figure 12 contains the resulting plot, which clearly visualizes the noisy data points.

In multiple mediator models, it can be of interest if the indirect effects are different from one another, or if they differ in magnitude (Hayes 2018, p.163–166). Function test_mediation() allows to make pairwise comparisons of indirect effects via the argument contrast. By setting this argument to "estimates", the pairwise differences of the estimates of the indirect effect are computed, whereas setting this argument to "absolute" yields the computation of

pairwise differences in absolute values. The output of test_mediation() then includes the estimates and confidence intervals for those contrasts, and also displays information on the definition of the contrasts.

```
R> set.seed(seed)
R> test mediation(f parallel, data = BSG2014, contrast = "absolute")
```

Robust bootstrap tests for indirect effects via regression

Indirect effects of x on y:

```
DataBootLowerUpperTotal0.241770.250380.1000350.4430ProceduralJustice0.079670.090430.0018160.2386InteractionalJustice0.162100.159950.0493380.2885Contrast-0.08243-0.06901-0.2242400.1235
```

Indirect effect contrast definition:

```
Label Definition
Contrast |ProceduralJustice| - |InteractionalJustice|
---
```

Level of confidence: 95 %

Number of bootstrap replicates: 5000

However, it is not actually necessary to run the entire bootstrap procedure again if a bootstrap test had already been performed without computing those contrasts. Function retest() allows to reanalyze the bootstrap estimates with different parameter settings, which saves computation time in such a case.

```
R> retest(robust_boot_parallel, contrast = "absolute")
```

Robust bootstrap tests for indirect effects via regression

Indirect effects of x on y:

```
        Data
        Boot
        Lower
        Upper

        Total
        0.24177
        0.25038
        0.100035
        0.4430

        ProceduralJustice
        0.07967
        0.09043
        0.001816
        0.2386

        InteractionalJustice
        0.16210
        0.15995
        0.049338
        0.2885

        Contrast
        -0.08243
        -0.06901
        -0.224240
        0.1235
```

Indirect effect contrast definition:

```
Label Definition

Contrast | Procedural Justice | - | Interactional Justice | ---
```

Level of confidence: 95 %

Number of bootstrap replicates: 5000

We emphasize that function retest() is available for computational convenience in case the analysis was by mistake conducted with the wrong parameter settings. It must not be abused for p hacking.

5. Summary and discussion

The R package **robmed** provides easy-to-use functionality for robust mediation analysis. It implements the robust bootstrap procedure of Alfons *et al.* (2022a), which yields reliable results under outliers and other deviations from normality assumptions, as well as diagnostic plots that allow to detect and further investigate such deviations. In addition, package **robmed** provides functionality for various other procedures for mediation analysis, as well as plots that allow to visualize and compare results from different methods. All implemented methods thereby share the same function interface and a clear class structure of the results. In particular, **robmed** introduces a new formula interface that allows to specify various types of mediation models with a single formula.

At present, there are some limitations of package **robmed**. First of all, the current version requires a numeric dependent variable and numeric mediators, although the independent variable and additional control variables may be binary or categorical. We aim to extend the fast-and-robust bootstrap methodology for robust estimators of logistic regression in order to add support for mediation analysis with binary dependent variables. Second, adding support for additional mediation models, such as moderated mediation and mediated moderation models (e.g., Muller et al. 2005) is planned for future versions. Third, the diagnostic plot of the regression weights would be even more useful if it included confidence bands, which could perhaps be constructed from the bootstrap procedure. Finally, a graphical user interface (GUI) could be beneficial for less proficient R users. Developing such a GUI, for instance as a web application based on package shiny (Chang, Cheng, Allaire, Sievert, Schloerke, Xie, Allen, McPherson, Dipert, and Borges 2021), is future work. In the meantime, we provide the R extension bundle **ROBMED** (Alfons 2022) for SPSS (IBM Corp. 2021), which links to the R package robmed and allows to use its main functionality through a GUI from within SPSS. Interested readers can obtain the extension bundle from https://github.com/ aalfons/ROBMED-RSPSS.

Computational details

The results in this paper were obtained using R version 4.5.1 (R Core Team 2022) with package **robmed** version 1.3.0 (Alfons 2025) and its dependencies **boot** version 1.3.32 (Canty and Ripley 2021), **ggplot2** version 4.0.0 (Wickham 2016), and **robustbase** version 0.99.6 (Maechler *et al.* 2022). Package **robmed** also builds upon packages **quantreg** (Koenker 2022) and **sn** (Azzalini 2022) for functionality not shown in this paper, as well as package **testthat** (Wickham 2011) for unit tests. R itself and all mentioned packages are available from the Comprehensive R Archive Network (CRAN) at https://CRAN.R-project.org/. The latest development version of package **robmed** can be obtained from https://github.com/aalfons/robmed.

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A. Details on the diagnostic plot with a tolerance ellipse

The diagnostic plot from function ellipse_plot() exploits the relationship between regression coefficients and the covariance matrix to draw a tolerance ellipse that illustrates how well the regression results represent the data. Our recommended robust bootstrap test for mediation analysis is based on a robust regression estimator that assigns robustness weights to the observations. Those robustness weights lie between 0 and 1, with lower weights indicating a higher degree of deviation. The corresponding tolerance ellipse is computed based on a weighted sample covariance matrix, using the weights returned by the robust regression. However, for such a plot to be meaningful, the weighted sample covariance matrix needs to yield a Fisher consistent estimator of the true covariance matrix under the model distribution.

The diagnostic plot is most useful if the data come from a multivariate elliptical distribution. Consider the regression model

$$Y = \alpha + \boldsymbol{X}^{\top} \boldsymbol{\beta} + \sigma \varepsilon,$$

In addition to the usual assumptions that $\varepsilon \sim N(0,1)$ and that X and ε are uncorrelated, we therefore also assume for the diagnostic plot that $X \sim N(\mu_X, \Sigma_{XX})$. We emphasize that this additional assumption is made only for the diagnostic plot, it is not required for the general applicability of our robust bootstrap test. Then we have $Z = (Y, X^{\top})^{\top} \sim N(\mu, \Sigma)$ with

$$\mu = \begin{pmatrix} \mu_Y \\ \mu_X \end{pmatrix}$$
 and $\Sigma = \begin{pmatrix} \Sigma_{YY} & \Sigma_{YX} \\ \Sigma_{XY} & \Sigma_{XX} \end{pmatrix}$.

Furthermore, the regression coefficients can be written as

$$\beta = \mathbf{\Sigma}_{XX}^{-1} \mathbf{\Sigma}_{XY},$$

$$\alpha = \mu_Y - \boldsymbol{\mu}_X^{\top} \boldsymbol{\beta}.$$

With observations $\mathbf{z}_i = (y_i, \mathbf{x}_i^{\top})^{\top}$ and with with robustness weights w_i from robust regression, i = 1, ..., n, we define the weighted center estimate \mathbf{m} and the weighted scatter matrix \mathbf{S} as

$$\mathbf{m} = \frac{1}{\sum_{i=1}^{n} w_i} \sum_{i=1}^{n} w_i \mathbf{z}_i,$$

$$\mathbf{S} = \frac{1}{(\sum_{i=1}^{n} w_i) - 1} \sum_{i=1}^{n} w_i (\mathbf{z}_i - \mathbf{m}) (\mathbf{z}_i - \mathbf{m})^{\top}.$$

Note that the denominator of S is chosen such that the weighted scatter matrix reduces to the unbiased sample covariance matrix if all observations receive full weight $w_i = 1$, i = 1, ..., n. Let F denote the cumulative distribution function (CDF) of the multivariate normal distribution $N(\mu, \Sigma)$ and let Φ denote the CDF of the univariate standard normal distribution N(0, 1). Then the functional forms of the estimators are given by

$$egin{aligned} m{m}(F) &= rac{1}{\delta} \int w(arepsilon) m{z} \ dF(m{z}), \ m{S}(F) &= rac{1}{\delta} \int w(arepsilon) (m{z} - m{m}(F)) (m{z} - m{m}(F))^{ op} \ dF(m{z}), \end{aligned}$$

where $\delta = \int w(\varepsilon) d\Phi(\varepsilon)$.

Keep in mind that we only consider robust regression with symmetric loss functions such that the weight function w is also symmetric, and let $\boldsymbol{m}(F) = (m_Y(F), \boldsymbol{m}_{\boldsymbol{X}}(F)^\top)^\top$. For the explanatory variables \boldsymbol{X} , we have

$$m_{\boldsymbol{X}}(F) = \frac{1}{\delta} \int w(\varepsilon) \boldsymbol{x} \, dF(\boldsymbol{z}) = \frac{1}{\delta} \underbrace{\int w(\varepsilon) \, d\Phi(\varepsilon)}_{=\delta} \int \boldsymbol{x} \, dF_{\boldsymbol{X}}(\boldsymbol{x}) = \boldsymbol{\mu}_{\boldsymbol{X}},$$

where $F_{\boldsymbol{X}}$ denotes the joint CDF of \boldsymbol{X} . To show that $m_Y(F) = \mu_Y$, we can assume without loss of generality that $\mu = (0, \dots, 0)^{\top}$ and that $\boldsymbol{\beta} = (0, \dots, 0)^{\top}$. Then $\alpha = 0$ and $\varepsilon = Y/\sigma$, and we need to show that $m_Y(F) = 0$. We obtain

$$m_Y(F) = \frac{1}{\delta} \int w(\varepsilon) y \, dF(\mathbf{z}) = \frac{1}{\delta} \int_{-\infty}^{\infty} \underbrace{w\left(\frac{y}{\sigma}\right) y f_Y(y)}_{=g(y)} \, dy, \tag{19}$$

where f_Y denotes the probability density function of the marginal distribution of Y. For the integrand in (19), it holds that g(-y) = -g(y), hence we have $m_Y(F) = 0$. Thus $\hat{\boldsymbol{\mu}} = \boldsymbol{m}$ is Fisher consistent for $\boldsymbol{\mu}$.

Since $0 \le w(\varepsilon) \le 1$, S(F) is expected to underestimate (some elements of) the covariance matrix Σ . However, for the submatrix involving only the explanatory variables X, we obtain

$$\begin{split} \boldsymbol{S}_{\boldsymbol{X}\boldsymbol{X}}(F) &= \frac{1}{\delta} \int w(\varepsilon) (\boldsymbol{x} - \underbrace{\boldsymbol{m}_{\boldsymbol{X}}(F)}) (\boldsymbol{x} - \underbrace{\boldsymbol{m}_{\boldsymbol{X}}(F)})^{\top} dF(\boldsymbol{z}) \\ &= \frac{1}{\delta} \underbrace{\int w(\varepsilon) d\Phi(\varepsilon)}_{=\delta} \int (\boldsymbol{x} - \boldsymbol{\mu}_{\boldsymbol{X}}) (\boldsymbol{x} - \boldsymbol{\mu}_{\boldsymbol{X}})^{\top} dF_{\boldsymbol{X}}(\boldsymbol{x}) = \boldsymbol{\Sigma}_{\boldsymbol{X}\boldsymbol{X}}. \end{split}$$

Thus

$$\hat{\boldsymbol{\Sigma}}_{\boldsymbol{X}\boldsymbol{X}} = \boldsymbol{S}_{\boldsymbol{X}\boldsymbol{X}} = \frac{1}{(\sum_{i=1}^{n} w_i) - 1} \sum_{i=1}^{n} w_i (\mathbf{x}_i - \bar{\mathbf{x}}) (\mathbf{x}_i - \bar{\mathbf{x}})^{\top}$$

is Fisher consistent for Σ_{XX} . With Fisher consistent estimators $\hat{\beta}$ and $\hat{\sigma}$ from robust regression, we can therefore construct a Fisher consistent estimator $\hat{\Sigma}$ of the covariance matrix Σ by computing

$$\begin{split} \mathbf{\hat{\Sigma}}_{XY} &= \mathbf{\hat{\Sigma}}_{XX} \mathbf{\hat{\beta}}, \\ \hat{\Sigma}_{YY} &= \mathbf{\hat{\beta}}^{\top} \mathbf{\hat{\Sigma}}_{XX} \mathbf{\hat{\beta}} + \hat{\sigma}^{2}, \\ \mathbf{\hat{\Sigma}} &= \begin{pmatrix} \hat{\Sigma}_{YY} & \mathbf{\hat{\Sigma}}_{XY}^{\top} \\ \mathbf{\hat{\Sigma}}_{XY} & \mathbf{\hat{\Sigma}}_{XX} \end{pmatrix}. \end{split}$$

B. Additional summary output of robust bootstrap tests

We first produce the summary of the robust bootstrap test for the serial multiple mediator model from Section 4.2. Since the diagnostic plot for this example is already shown in Figure 11, we set the argument plot = FALSE to suppress the diagnostic plot.

R> summary(robust_boot_serial, plot = FALSE)

Robust bootstrap tests for indirect effects via regression

Serial multiple mediator model

x = ValueDiversity

y = TeamScore

m1 = TaskConflict

m2 = TeamCommitment

Sample size: 89

Outcome variable: TaskConflict

Coefficients:

Data Boot Std. Error z value Pr(>|z|)

(Intercept) 1.1182 1.1160 0.1808 6.174 6.68e-10 ***
ValueDiversity 0.3197 0.3220 0.1083 2.973 0.00294 **

Robust residual standard error: 0.3033 on 87 degrees of freedom Robust R-squared: 0.1181, Adjusted robust R-squared: 0.108

Robust F-statistic: 9.113 on 1 and Inf DF, p-value: 0.002539

Robustness weights:

4 observations are potential outliers with weight \leftarrow 1.3e-05:

[1] 48 58 76 79

Outcome variable: TeamCommitment

Coefficients:

Data Boot Std. Error z value Pr(>|z|)

(Intercept) 4.54771 4.54858 0.29539 15.399 < 2e-16 *** TaskConflict -0.35750 -0.36100 0.13090 -2.758 0.00582 ** ValueDiversity 0.04233 0.04560 0.09974 0.457 0.64750

Robust residual standard error: 0.3842 on 86 degrees of freedom

Robust R-squared: 0.1155, Adjusted robust R-squared: 0.09492

Robust F-statistic: 2.233 on 2 and Inf DF, p-value: 0.1072

Robustness weights:

No potential outliers with weight < 0.0011 detected.

The minimum weight is 0.15.

Outcome variable: TeamScore

Coefficients:

```
Boot Std. Error z value Pr(>|z|)
                 Data
              50.3061 49.7823
                                 10.3622
                                           4.804 1.55e-06 ***
(Intercept)
TaskConflict
                                  2.4360
                                           0.637
                1.6315 1.5517
                                                   0.524
TeamCommitment 11.0984 11.1876
                                  2.1851
                                           5.120 3.06e-07 ***
ValueDiversity 0.3622 0.5341
                                  2.6841
                                           0.199
                                                    0.842
Robust residual standard error: 8.221 on 85 degrees of freedom
Robust R-squared: 0.2317,
                                 Adjusted robust R-squared:
Robust F-statistic: 2.261 on 3 and Inf DF, p-value: 0.07914
Robustness weights:
3 observations are potential outliers with weight <= 0:
[1] 10 32 38
Total effect of x on y:
                  Data
                         Boot Std. Error z value Pr(>|z|)
ValueDiversity 0.08503 0.27541
                                 2.95979
                                           0.093
                                                    0.926
Direct effect of x on y:
                Data
                       Boot Std. Error z value Pr(>|z|)
ValueDiversity 0.3622 0.5341
                                2.6841
                                         0.199
                                                  0.842
Indirect effects of x on y:
                    Boot Lower
                                  Upper
             Data
Total
         -0.2772 -0.2587 -3.430 2.6489
Indirect1 0.5216 0.5193 -1.130 2.3905
Indirect2 0.4698 0.5363 -1.594 3.0285
Indirect3 -1.2686 -1.3142 -3.140 -0.2413
Indirect effect paths:
 Label
          Path
 Indirect1 ValueDiversity -> TaskConflict
                                           -> TeamScore
 Indirect2 ValueDiversity -> TeamCommitment -> TeamScore
 Indirect3 ValueDiversity -> TaskConflict
                                           -> TeamCommitment -> TeamScore
Level of confidence: 95 %
Number of bootstrap replicates: 5000
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Finally, we display the summary of the robust bootstrap test for the parallel multiple mediator model with control variables from Section 4.3. Figure 13 contains the resulting diagnostic plot.

```
R> summary(robust_boot_parallel)
```

Robust bootstrap tests for indirect effects via regression

Parallel multiple mediator model

x = SharedLeadership

y = TeamPerformance

m1 = ProceduralJustice

m2 = InteractionalJustice

Covariates:

[1] AgeDiversity GenderDiversity

Sample size: 89

Outcome variable: ProceduralJustice

Coefficients:

	Data	Boot	Std. Error	z value	Pr(> z)	
(Intercept)	3.37132	3.34831	0.22055	15.181	<2e-16	***
${\tt SharedLeadership}$	0.12575	0.13409	0.07246	1.851	0.0642	
AgeDiversity	0.05232	0.04994	0.03030	1.649	0.0992	
GenderDiversity	0.22030	0.20938	0.14887	1.406	0.1596	

Robust residual standard error: 0.2377 on 85 degrees of freedom
Robust R-squared: 0.1277, Adjusted robust R-squared: 0.09696
Robust F-statistic: 1.208 on 3 and Inf DF, p-value: 0.3052

Robustness weights:

No potential outliers with weight < 0.0011 detected.

The minimum weight is 0.27.

Outcome variable: InteractionalJustice

Coefficients:

Robust residual standard error: 0.2788 on 85 degrees of freedom Robust R-squared: 0.2847, Adjusted robust R-squared: 0.2595 Robust F-statistic: 2.169 on 3 and Inf DF, p-value: 0.08934

Robustness weights:

2 observations are potential outliers with weight <= 0:
[1] 31 57</pre>

Outcome variable: TeamPerformance

Coefficients:

	Data	Boot	Std. Error	z value	Pr(> z)	
(Intercept)	-0.800757	-0.814795	0.600424	-1.357	0.1748	
ProceduralJustice	0.633597	0.649305	0.200495	3.239	0.0012	**
InteractionalJustice	0.535426	0.523275	0.172359	3.036	0.0024	**
SharedLeadership	-0.032971	-0.030379	0.100631	-0.302	0.7627	
AgeDiversity	0.002538	0.001760	0.036480	0.048	0.9615	
GenderDiversity	0.275618	0.271119	0.154746	1.752	0.0798	

Robust residual standard error: 0.2558 on 83 degrees of freedom
Robust R-squared: 0.5988, Adjusted robust R-squared: 0.5746
Robust F-statistic: 3.476 on 5 and Inf DF, p-value: 0.003833

Robustness weights:

3 observations are potential outliers with weight <= 0:
[1] 33 38 79</pre>

Total effect of x on y:

Data Boot Std. Error z value Pr(>|z|) SharedLeadership 0.2088 0.2200 0.1049 2.097 0.036 *

Direct effect of x on y:

Indirect effects of x on y:

 Data
 Boot
 Lower
 Upper

 Total
 0.24177
 0.25038
 0.100035
 0.4430

 ProceduralJustice
 0.07967
 0.09043
 0.001816
 0.2386

 InteractionalJustice
 0.16210
 0.15995
 0.049338
 0.2885

Level of confidence: 95 %

Number of bootstrap replicates: 5000

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

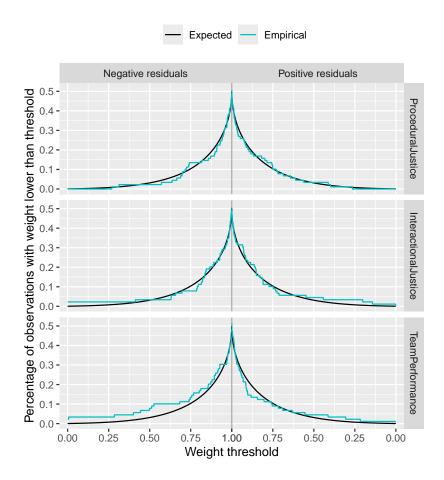


Figure 13: Diagnostic plot of the regression weights from the robust bootstrap procedure of Alfons $et\ al.\ (2022a)$ in the example for a parallel multiple mediator model.

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