lmer for SAS PROC MIXED Users

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1 Introduction

The `lmer` function from the `lme4` package for R is used to fit linear mixed-effects models. It is similar in scope to the SAS procedure `PROC MIXED` described in Littell et˜al. (1996).

A file on the SAS Institute web site (http://www.sas.com) contains all the data sets in the book and all the SAS programs used in Littell et˜al. (1996). We have converted the data sets from the tabular representation used for SAS to the `data.frame` objects used by `lmer`. To help users familiar with `SAS PROC MIXED` get up to speed with `lmer` more quickly, we provide transcripts of some `lmer` analyses paralleling the `SAS PROC MIXED` analyses in Littell et˜al. (1996).

In this paper we highlight some of the similarities and differences of `lmer` analysis and `SAS PROC MIXED` analysis.

2 Similarities between lmer and SAS PROC MIXED

Both `SAS PROC MIXED` and `lmer` can fit linear mixed-effects models expressed in the Laird-Ware formulation. For a single level of grouping Laird and Ware (1982) write the n_i-dimensional response vector $\boldsymbol{y}_i$ for the ith experimental

unit as

$$\begin{aligned} \boldsymbol{y}_i &= \boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{Z}_i\boldsymbol{b}_i + \boldsymbol{\epsilon}_i, \quad i = 1, \ldots, M \\ \boldsymbol{b}_i &\sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}), \quad \boldsymbol{\epsilon}_i \sim \mathcal{N}(\boldsymbol{0}, \sigma^2\boldsymbol{I}) \end{aligned} \tag{1}$$

where $\boldsymbol{\beta}$ is the p-dimensional vector of *fixed effects*, $\boldsymbol{b}_i$ is the q-dimensional vector of *random effects*, $\boldsymbol{X}_i$ (of size $n_i \times p$) and $\boldsymbol{Z}_i$ (of size $n_i \times q$) are known fixed-effects and random-effects regressor matrices, and $\boldsymbol{\epsilon}_i$ is the n_i-dimensional *within-group error* vector with a spherical Gaussian distribution. The assumption $\mathrm{Var}(\boldsymbol{\epsilon}_i) = \sigma^2\boldsymbol{I}$ can be relaxed using additional arguments in the model fitting.

The basic specification of the model requires a linear model expression for the fixed effects and a linear model expression for the random effects. In `SAS PROC MIXED` the fixed-effects part is specified in the `model` statement and the random-effects part in the `random` statement. In `lmer` the fixed effects and the random effects are both specified as terms in the `formula` argument to `lmer`.

Both `SAS PROC MIXED` and `lmer` allow a mixed-effects model to be fit by maximum likelihood (`method = ml` in SAS) or by maximum residual likelihood, sometimes also called restricted maximum likelihood or REML. This is the default criterion in `SAS PROC MIXED` and in `lmer`. To get ML estimates use the optional argument `REML=FALSE` in the call to `lmer`.

3 Important differences

The output from `PROC MIXED` typically includes values of the Akaike Information Criterion (AIC) and Schwartz's Bayesian Criterion (SBC). These are used to compare different models fit to the same data. The output of the `summary` function applied to the object created by `lmer` also produces values of AIC and BIC but the definitions used in older versions of `PROC MIXED` are different from those used in more recent versions of `PROC MIXED` and in `lmer`. In `lmer` the definitions are such that "smaller is better". In some older versions of `PROC MIXED` the definitions are such that "bigger is better".

When models are fit by REML, the values of AIC, SBC (or BIC) and the log-likelihood can only be compared between models with exactly the same fixed-effects structure. When models are fit by maximum likelihood these criteria can be compared between any models fit to the same data. That is,

these quality-of-fit criteria can be used to evaluate different fixed-effects specifications or different random-effects specifications or different specifications of both fixed effects and random effects.

We encourage developing and testing the model using likelihood ratio tests or the AIC and BIC criteria. Once a form for both the random effects and the fixed effects has been determined, the model can be refit with `REML = TRUE` if the restricted estimates of the variance components are desired. Note that the `update` function provides a convenient way of refitting a model with changes to one or more arguments.

4 Data manipulation

Both `PROC MIXED` and `lmer` work with data in a tabular form with one row per observation. There are, however, important differences in the internal representations of variables in the data.

In SAS a qualitative factor can be stored either as numerical values or alphanumeric labels. When a factor stored as numerical values is used in `PROC MIXED` it is listed in the `class` statement to indicate that it is a factor. In S this information is stored with the data itself by converting the variable to a factor when it is first stored. If the factor represents an ordered set of levels, it should be converted to an `ordered` factor.

For example the SAS code

```
data animal;
 input trait animal y;
 datalines;
1 1 6
1 2 8
1 3 7
2 1 9
2 2 5
2 3 .
;
```

would require that the `trait` and `animal` variables be specified in a class statement in any model that is fit.

In R these data could be read from a file, say `animal.dat`, and converted to factors by

```
animal <- within(read.table("animal.dat", header = TRUE),
                 {
```

```
    trait <- factor(trait)
    animal <- factor(animal)
})
```

In general it is a good idea to check the types of variables in a data frame before working with it. One way of doing this is to apply the function data.class to each variable in turn using the sapply function.

```
> sapply(Animal, data.class)
        Sire          Dam AvgDailyGain
    "factor"     "factor"    "numeric"
> str(Animal)
'data.frame':        20 obs. of  3 variables:
 $ Sire        : Factor w/ 5 levels "1","2","3","4",..: 1 1 1 1 2 2 2 2 3 3 .
 $ Dam         : Factor w/ 2 levels "1","2": 1 1 2 2 1 1 2 2 1 1 ...
 $ AvgDailyGain: num  2.24 1.85 2.05 2.41 1.99 1.93 2.72 2.32 2.33 2.68 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 AvgDailyGain ~ 1 | Sire/Dam
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ order.groups:List of 2
  .. ..$ Sire: logi TRUE
  .. ..$ Dam : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      :List of 1
  .. ..$ AvgDailyGain: chr "Average Daily Weight Gain"
  ..$ units       : list()
```

4.1 Unique levels of factors

Designs with nested grouping factors are indicated differently in the two languages. An example of such an experimental design is the semiconductor experiment described in section 2.2 of Littell et˜al. (1996) where twelve wafers are assigned to four experimental treatments with three wafers per treatment. The levels for the wafer factor are 1, 2, and 3 but the wafer factor is only meaningful within the same level of the treatment factor, et. There is nothing associating wafer 1 of the third treatment group with wafer 1 of the first treatment group.

In SAS this nesting of factors is denoted by wafer(et). In S the nesting is written with ~ ET/Wafer and read "wafer within ET". If both levels of

nested factors are to be associated with random effects then this is all you need to know. You would use an expression with a `"/"` in the grouping factor part of the formula in the call to `lmer` object. The random effects term would be either

```
(1 | ET/Wafer)
```

or, equivalently

```
(1 | ET:Wafer) + (1 | ET)
```

In this case, however, there would not usually be any random effects associated with the "experimental treatment" or `ET` factor. The only random effects are at the `Wafer` level. It is necessary to create a factor that will have unique levels for each `Wafer` within each level of `ET`. One way to do this is to assign

```
> Semiconductor <- within(Semiconductor, Grp <- factor(ET:Wafer))
```

after which we could specify a random effects term of `(1 | Grp)`. Alternatively, we can use the explicit term

```
(1 | ET:Wafer)
```

4.2 General approach

As a general approach to importing data into R for mixed-effects analysis you should:

- Create a `data.frame` with one row per observation and one column per variable.
- Use `factor` or `as.factor` to explicitly convert any ordered factors to class `ordered`.
- Use `ordered` or `as.ordered` to explicitly convert any ordered factors to class `ordered`.
- If necessary, use interaction terms to create a factor with unique levels from inner nested factors.
- Plot the data. Plot it several ways. The use of lattice graphics is closely integrated with the `lme4` library. Lattice plots can provide invaluable insight into the structure of the data. Use them.

5 Contrasts

When comparing estimates produced by `SAS PROC MIXED` and by `lmer` one must be careful to consider the contrasts that are used to define the effects of factors. In SAS a model with an intercept and a qualitative factor is defined in terms of the intercept and the indicator variables for all but the last level of the factor. The default behaviour in S is to use the Helmert contrasts for the factor. On a balanced factor these provide a set of orthogonal contrasts. In R the default is the "treatment" contrasts which are almost the same as the SAS parameterization except that they drop the indicator of the first level, not the last level.

When in doubt, check which contrasts are being used with the contrasts function.

To make comparisons easier, you may find it worthwhile to declare

```
> options(contrasts = c(factor = "contr.SAS", ordered = "contr.poly"))
```

at the beginning of your session.

References

Nan˜M. Laird and James˜H. Ware. Random-effects models for longitudinal data. *Biometrics*, 38:963–974, 1982.

Ramon˜C. Littell, George˜A. Milliken, Walter˜W. Stroup, and Russell˜D. Wolfinger. *SAS System for Mixed Models*. SAS Institute, Inc., 1996.

A AvgDailyGain

```
> print(xyplot(adg ~ Treatment | Block, AvgDailyGain, type = c("g", "p", "r")
+         xlab = "Treatment (amount of feed additive)",
+         ylab = "Average daily weight gain (lb.)", aspect = "xy",
+         index.cond = function(x, y) coef(lm(y ~ x))[1]))

> ## compare with output 5.1, p. 178
> (fm1Adg <- lmer(adg ~ (Treatment - 1)*InitWt + (1 | Block), AvgDailyGain))
Linear mixed model fit by REML ['lmerMod']
Formula: adg ~ (Treatment - 1) * InitWt + (1 | Block)
   Data: AvgDailyGain
```

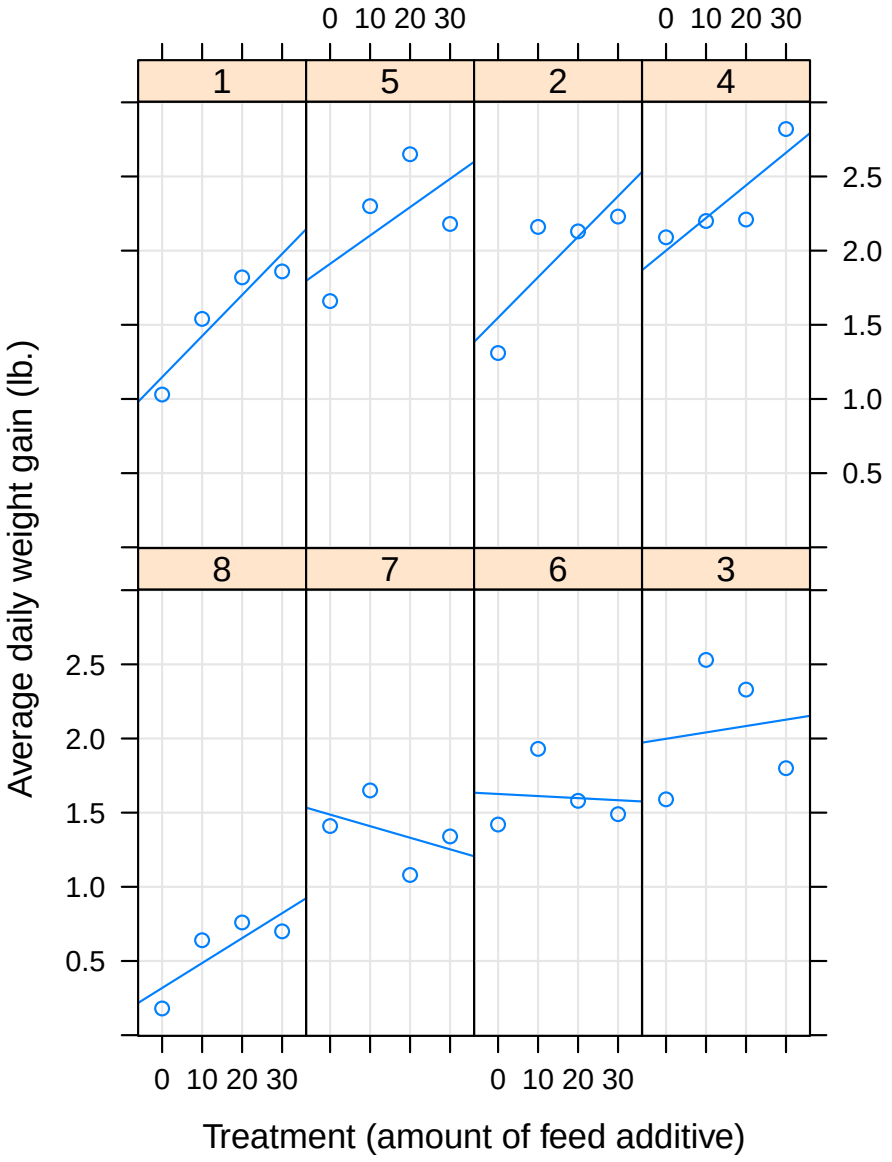

Figure 1: Average daily weight gain

```
REML criterion at convergence: 65.3268
Random effects:
 Groups   Name        Std.Dev.
 Block    (Intercept) 0.5092
 Residual             0.2223
Number of obs: 32, groups: Block, 8
Fixed Effects:
        Treatment0         Treatment10         Treatment20
          0.439137            1.426118            0.479628
       Treatment30              InitWt   Treatment0:InitWt
          0.200107            0.004448           -0.002154
Treatment10:InitWt  Treatment20:InitWt
         -0.003365           -0.001082
> anova(fm1Adg)   # checking significance of terms
Analysis of Variance Table
                Df Sum Sq Mean Sq F value
Treatment        4 5.7248 1.43119 28.9543
InitWt           1 0.5495 0.54953 11.1175
Treatment:InitWt 3 0.1381 0.04603  0.9312
> ## common slope model
> (fm2Adg <- lmer(adg ~ InitWt + Treatment + (1 | Block), AvgDailyGain))
Linear mixed model fit by REML ['lmerMod']
Formula: adg ~ InitWt + Treatment + (1 | Block)
   Data: AvgDailyGain
REML criterion at convergence: 36.3373
Random effects:
 Groups   Name        Std.Dev.
 Block    (Intercept) 0.4908
 Residual             0.2238
Number of obs: 32, groups: Block, 8
Fixed Effects:
(Intercept)       InitWt   Treatment0  Treatment10  Treatment20
    0.80111      0.00278     -0.55207     -0.06857     -0.08813
> anova(fm2Adg)
Analysis of Variance Table
          Df  Sum Sq Mean Sq F value
InitWt     1 0.51455 0.51455  10.275
Treatment  3 1.52670 0.50890  10.162
> (fm3Adg <- lmer(adg ~ InitWt + Treatment - 1 + (1 | Block), AvgDailyGain))
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: adg ~ InitWt + Treatment - 1 + (1 | Block)
   Data: AvgDailyGain
REML criterion at convergence: 36.3373
Random effects:
 Groups   Name        Std.Dev.
 Block    (Intercept) 0.4908
 Residual             0.2238
Number of obs: 32, groups: Block, 8
Fixed Effects:
     InitWt   Treatment0  Treatment10  Treatment20  Treatment30
    0.00278      0.24903      0.73254      0.71298      0.80111
```

B BIB

```
> print(xyplot(y ~ x | Block, BIB, groups = Treatment, type = c("g", "p"),
+              aspect = "xy", auto.key = list(points = TRUE, space = "right",
+              lines = FALSE)))

> ## compare with Output 5.7, p. 188
> (fm1BIB <- lmer(y ~ Treatment * x + (1|Block), BIB))
Linear mixed model fit by REML ['lmerMod']
Formula: y ~ Treatment * x + (1 | Block)
   Data: BIB
REML criterion at convergence: 104.8945
Random effects:
 Groups   Name        Std.Dev.
 Block    (Intercept) 4.272
 Residual             1.096
Number of obs: 24, groups: Block, 8
Fixed Effects:
 (Intercept)    Treatment1    Treatment2    Treatment3             x
    22.36784       4.42949      -0.43737       6.27864       0.44255
Treatment1:x  Treatment2:x  Treatment3:x
    -0.22377       0.05338      -0.17918
> anova(fm1BIB)     # strong evidence of different slopes
Analysis of Variance Table
            Df  Sum Sq Mean Sq  F value
Treatment    3  23.447   7.816   6.5110
x            1 136.809 136.809 113.9695
Treatment:x  3  18.427   6.142   5.1169
```

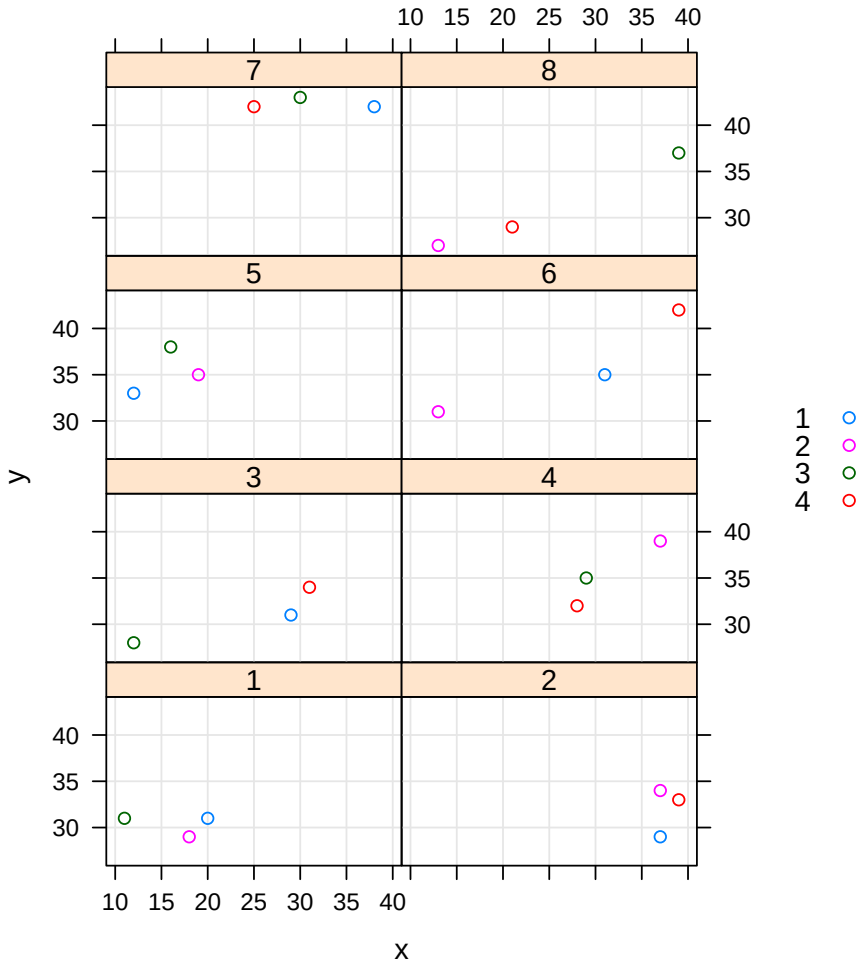

Figure 2: Balanced incomplete block design

```
> ## compare with Output 5.9, p. 193
> (fm2BIB <- lmer(y ~ Treatment + x:Grp + (1|Block), BIB))
Linear mixed model fit by REML ['lmerMod']
Formula: y ~ Treatment + x:Grp + (1 | Block)
   Data: BIB
REML criterion at convergence: 99.177
Random effects:
 Groups   Name        Std.Dev.
 Block    (Intercept) 4.304
 Residual             1.019
Number of obs: 24, groups: Block, 8
Fixed Effects:
(Intercept)   Treatment1   Treatment2   Treatment3      x:Grp13
    20.9452       5.3414       1.1356       8.1810       0.2395
    x:Grp24
     0.4892
> anova(fm2BIB)
Analysis of Variance Table
          Df  Sum Sq Mean Sq F value
Treatment  3  23.424   7.808  7.5236
x:Grp      2 154.733  77.367 74.5471
```

C Bond

```
> ## compare with output 1.1 on p. 6
> (fm1Bond <- lmer(pressure ~ Metal + (1|Ingot), Bond))
Linear mixed model fit by REML ['lmerMod']
Formula: pressure ~ Metal + (1 | Ingot)
   Data: Bond
REML criterion at convergence: 107.7902
Random effects:
 Groups   Name        Std.Dev.
 Ingot    (Intercept) 3.383
 Residual             3.220
Number of obs: 21, groups: Ingot, 7
Fixed Effects:
(Intercept)       Metalc       Metali
    71.1000      -0.9143       4.8000
> anova(fm1Bond)
Analysis of Variance Table
      Df Sum Sq Mean Sq F value
Metal  2  131.9   65.95  6.3587
```

D Cultivation

```
> str(Cultivation)
'data.frame':        24 obs. of  4 variables:
 $ Block: Factor w/ 4 levels "1","2","3","4": 1 1 1 1 1 1 2 2 2 2 ...
 $ Cult : Factor w/ 2 levels "a","b": 1 1 1 2 2 2 1 1 1 2 ...
 $ Inoc : Factor w/ 3 levels "con","dea","liv": 1 2 3 1 2 3 1 2 3 1 ...
 $ drywt: num  27.4 29.7 34.5 29.4 32.5 34.4 28.9 28.7 33.4 28.7 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 drywt ~ 1 | Block/Cult
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ order.groups:List of 2
  .. ..$ Block: logi TRUE
  .. ..$ Cult : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       :List of 1
  .. ..$ Cult:Class 'formula' length 2 ~Inoc
  .. .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ labels      :List of 1
  .. ..$ drywt: chr "Yield"
  ..$ units       : list()
> xtabs(~Block+Cult, Cultivation)
     Cult
Block a b
    1 3 3
    2 3 3
    3 3 3
    4 3 3
> (fm1Cult <- lmer(drywt ~ Inoc * Cult + (1|Block) + (1|Cult), Cultivation))
Linear mixed model fit by REML ['lmerMod']
Formula: drywt ~ Inoc * Cult + (1 | Block) + (1 | Cult)
   Data: Cultivation
REML criterion at convergence: 68.4874
Random effects:
 Groups   Name        Std.Dev.
 Block    (Intercept) 1.099
 Cult     (Intercept) 1.105
 Residual             1.094
Number of obs: 24, groups: Block, 4; Cult, 2
Fixed Effects:
```

```
  (Intercept)        Inoccon        Inocdea          Culta  Inoccon:Culta
       33.525         -5.500         -2.875         -0.375          0.250
Inocdea:Culta
       -1.025
> anova(fm1Cult)
Analysis of Variance Table
          Df  Sum Sq Mean Sq F value
Inoc       2 118.176  59.088 49.3908
Cult       1   0.182   0.182  0.1517
Inoc:Cult  2   1.826   0.913  0.7631
> (fm2Cult <- lmer(drywt ~ Inoc + Cult + (1|Block) + (1|Cult), Cultivation))
Linear mixed model fit by REML ['lmerMod']
Formula: drywt ~ Inoc + Cult + (1 | Block) + (1 | Cult)
   Data: Cultivation
REML criterion at convergence: 73.7535
Random effects:
 Groups   Name        Std.Dev.
 Block    (Intercept) 1.101
 Cult     (Intercept) 1.070
 Residual             1.078
Number of obs: 24, groups: Block, 4; Cult, 2
Fixed Effects:
(Intercept)      Inoccon      Inocdea        Culta
    33.6542      -5.3750      -3.3875      -0.6333
> anova(fm2Cult)
Analysis of Variance Table
     Df  Sum Sq Mean Sq F value
Inoc  2 118.176  59.088 50.8069
Cult  1   0.188   0.188  0.1616
> (fm3Cult <- lmer(drywt ~ Inoc + (1|Block) + (1|Cult), Cultivation))
Linear mixed model fit by REML ['lmerMod']
Formula: drywt ~ Inoc + (1 | Block) + (1 | Cult)
   Data: Cultivation
REML criterion at convergence: 75.6778
Random effects:
 Groups   Name        Std.Dev.
 Block    (Intercept) 1.1013
 Cult     (Intercept) 0.3219
 Residual             1.0784
Number of obs: 24, groups: Block, 4; Cult, 2
Fixed Effects:
(Intercept)      Inoccon      Inocdea
     33.338       -5.375       -3.388
```

```
> anova(fm3Cult)
Analysis of Variance Table
     Df Sum Sq Mean Sq F value
Inoc  2 118.18  59.088  50.807
```

E Demand

```
> ## compare to output 3.13, p. 132
> (fm1Demand <-
+  lmer(log(d) ~ log(y) + log(rd) + log(rt) + log(rs) + (1|State) + (1|Year),
+       Demand))
Linear mixed model fit by REML ['lmerMod']
Formula: log(d) ~ log(y) + log(rd) + log(rt) + log(rs) + (1 | State) +      (
   Data: Demand
REML criterion at convergence: -240.1653
Random effects:
 Groups   Name        Std.Dev.
 Year     (Intercept) 0.01627
 State    (Intercept) 0.17177
 Residual             0.03342
Number of obs: 77, groups: Year, 11; State, 7
Fixed Effects:
(Intercept)       log(y)      log(rd)      log(rt)      log(rs)
   -1.28382      1.06978     -0.29533      0.03988     -0.32673
```

F HR

```
> ## linear trend in time
> (fm1HR <- lmer(HR ~ Time * Drug + baseHR + (Time|Patient), HR))
Linear mixed model fit by REML ['lmerMod']
Formula: HR ~ Time * Drug + baseHR + (Time | Patient)
   Data: HR
REML criterion at convergence: 767.607
Random effects:
 Groups   Name        Std.Dev. Corr
 Patient  (Intercept) 7.787
          Time        6.147    -0.56
 Residual             4.936
Number of obs: 120, groups: Patient, 24
Fixed Effects:
```

```
(Intercept)          Time         Druga         Drugb        baseHR
    33.9776      -3.1970        3.5992        7.0912        0.5434
 Time:Druga   Time:Drugb
    -7.5013      -3.9894
> anova(fm1HR)
Analysis of Variance Table
          Df Sum Sq Mean Sq F value
Time       1 379.23  379.23 15.5671
Drug       2  92.88   46.44  1.9064
baseHR     1 533.27  533.27 21.8905
Time:Drug  2  72.12   36.06  1.4802
> ## remove interaction
> (fm3HR <- lmer(HR ~ Time + Drug + baseHR + (Time|Patient), HR))
Linear mixed model fit by REML ['lmerMod']
Formula: HR ~ Time + Drug + baseHR + (Time | Patient)
   Data: HR
REML criterion at convergence: 779.8283
Random effects:
 Groups   Name        Std.Dev. Corr
 Patient  (Intercept) 7.846
          Time        6.400    -0.57
 Residual             4.936
Number of obs: 120, groups: Patient, 24
Fixed Effects:
(Intercept)          Time         Druga         Drugb        baseHR
    36.0463      -7.0273       -0.4524        4.9365        0.5434
> anova(fm3HR)
Analysis of Variance Table
       Df Sum Sq Mean Sq F value
Time    1 364.02  364.02 14.9431
Drug    2  92.88   46.44  1.9064
baseHR  1 533.27  533.27 21.8905
> ## remove Drug term
> (fm4HR <- lmer(HR ~ Time + baseHR + (Time|Patient), HR))
Linear mixed model fit by REML ['lmerMod']
Formula: HR ~ Time + baseHR + (Time | Patient)
   Data: HR
REML criterion at convergence: 791.1481
Random effects:
 Groups   Name        Std.Dev. Corr
```

```
 Patient  (Intercept) 7.939
          Time        6.400    -0.55
 Residual             4.936
Number of obs: 120, groups: Patient, 24
Fixed Effects:
(Intercept)          Time       baseHR
    36.9313      -7.0273       0.5508
> anova(fm4HR)
Analysis of Variance Table
       Df Sum Sq Mean Sq F value
Time    1 364.03  364.03  14.943
baseHR  1 534.87  534.87  21.956
```

G Mississippi

```
> ## compare with output 4.1, p. 142
> (fm1Miss <- lmer(y ~ 1 + (1 | influent), Mississippi))
Linear mixed model fit by REML ['lmerMod']
Formula: y ~ 1 + (1 | influent)
   Data: Mississippi
REML criterion at convergence: 252.3511
Random effects:
 Groups   Name        Std.Dev.
 influent (Intercept) 7.958
 Residual             6.531
Number of obs: 37, groups: influent, 6
Fixed Effects:
(Intercept)
      21.22
> ## compare with output 4.2, p. 143
> (fm1MLMiss <- lmer(y ~ 1 + (1 | influent), Mississippi, REML=FALSE))
Linear mixed model fit by maximum likelihood ['lmerMod']
Formula: y ~ 1 + (1 | influent)
   Data: Mississippi
      AIC       BIC    logLik  deviance
 262.5570  267.3898 -128.2785  256.5570
Random effects:
 Groups   Name        Std.Dev.
 influent (Intercept) 7.159
 Residual             6.534
```

```
Number of obs: 37, groups: influent, 6
Fixed Effects:
(Intercept)
      21.22
> ranef(fm1MLMiss)          # BLUP's of random effects on p. 144
$influent
  (Intercept)
1   0.3097833
2  -6.5772239
3  -3.7862717
4   2.8826693
5  -5.8435163
6  13.0145592

attr(,"class")
[1] "ranef.mer"
> ranef(fm1Miss)            # BLUP's of random effects on p. 142
$influent
  (Intercept)
1    0.309286
2   -6.719325
3   -3.897940
4    2.946101
5   -6.012976
6   13.374854

attr(,"class")
[1] "ranef.mer"
> VarCorr(fm1Miss)          # compare to output 4.7, p. 148
 Groups   Name        Std.Dev.
 influent (Intercept) 7.9576
 Residual             6.5313
> ## compare to output 4.8 and 4.9, pp. 150-152
> (fm2Miss <- lmer(y ~ Type + (1 | influent), Mississippi))
Linear mixed model fit by REML ['lmerMod']
Formula: y ~ Type + (1 | influent)
   Data: Mississippi
REML criterion at convergence: 234.5246
Random effects:
 Groups   Name        Std.Dev.
```

```
 influent (Intercept) 3.869
 Residual             6.520
Number of obs: 37, groups: influent, 6
Fixed Effects:
(Intercept)        Type1        Type2
      36.40       -20.80       -16.46
> anova(fm2Miss)
Analysis of Variance Table
     Df Sum Sq Mean Sq F value
Type  2 541.75  270.88  6.3715
```

H Multilocation

```
> str(Multilocation)
'data.frame':        108 obs. of  7 variables:
 $ obs     : num  3 4 6 7 9 10 12 16 19 20 ...
 $ Location: Factor w/ 9 levels "A","B","C","D",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ Block   : Factor w/ 3 levels "1","2","3": 1 1 1 1 2 2 2 2 3 3 ...
 $ Trt     : Factor w/ 4 levels "1","2","3","4": 3 4 2 1 2 1 3 4 1 2 ...
 $ Adj     : num  3.16 3.12 3.16 3.25 2.71 ...
 $ Fe      : num  7.1 6.68 6.83 6.53 8.25 ...
 $ Grp     : Factor w/ 27 levels "A/1","A/2","A/3",..: 1 1 1 1 2 2 2 2 3 3 ..
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 Adj ~ 1 | Location/Block
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ order.groups:List of 2
  .. ..$ Location: logi TRUE
  .. ..$ Block   : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       :List of 1
  .. ..$ Block:Class 'formula' length 2 ~Trt
  .. .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ labels      :List of 1
  .. ..$ Adj: chr "Adjusted yield"
  ..$ units       : list()
> ### Create a Block %in% Location factor
> Multilocation$Grp <- with(Multilocation, Block:Location)
> (fm1Mult <- lmer(Adj ~ Location * Trt + (1|Grp), Multilocation))
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: Adj ~ Location * Trt + (1 | Grp)
   Data: Multilocation
REML criterion at convergence: 10.6462
Random effects:
 Groups   Name        Std.Dev.
 Grp      (Intercept) 0.07496
 Residual             0.18595
Number of obs: 108, groups: Grp, 27
Fixed Effects:
   (Intercept)       LocationA       LocationB       LocationC
       2.35923         0.64930         0.06643         0.54533
     LocationD       LocationE       LocationF       LocationG
       0.37413         0.55000         0.99810         0.36057
     LocationH            Trt1            Trt2            Trt3
       1.01403         0.22720        -0.00140         0.42323
LocationA:Trt1  LocationB:Trt1  LocationC:Trt1  LocationD:Trt1
      -0.18853        -0.27523        -0.04000        -0.53513
LocationE:Trt1  LocationF:Trt1  LocationG:Trt1  LocationH:Trt1
      -0.26297        -0.27153         0.20323        -0.14953
LocationA:Trt2  LocationB:Trt2  LocationC:Trt2  LocationD:Trt2
      -0.09347        -0.32273         0.08960        -0.29693
LocationE:Trt2  LocationF:Trt2  LocationG:Trt2  LocationH:Trt2
      -0.30693        -0.30993        -0.10860        -0.33060
LocationA:Trt3  LocationB:Trt3  LocationC:Trt3  LocationD:Trt3
      -0.40247        -0.56550        -0.12247        -0.54840
LocationE:Trt3  LocationF:Trt3  LocationG:Trt3  LocationH:Trt3
      -0.32863        -0.46257        -0.25297        -0.37203
> anova(fm1Mult)
Analysis of Variance Table
             Df Sum Sq Mean Sq F value
Location      8 6.9476 0.86845 25.1150
Trt           3 1.2217 0.40725 11.7774
Location:Trt 24 0.9966 0.04152  1.2008
> (fm2Mult <- lmer(Adj ~ Location + Trt + (1|Grp), Multilocation))
Linear mixed model fit by REML ['lmerMod']
Formula: Adj ~ Location + Trt + (1 | Grp)
   Data: Multilocation
REML criterion at convergence: -6.0011
Random effects:
```

```
 Groups   Name        Std.Dev.
 Grp      (Intercept) 0.07131
 Residual             0.19161
Number of obs: 108, groups: Grp, 27
Fixed Effects:
(Intercept)    LocationA    LocationB    LocationC    LocationD
    2.53296      0.47818     -0.22443      0.52712      0.02902
  LocationE    LocationF    LocationG    LocationH         Trt1
    0.32537      0.73709      0.32098      0.80099      0.05834
       Trt2         Trt3
   -0.18802      0.08379
> (fm3Mult <- lmer(Adj ~ Location       + (1|Grp), Multilocation))
Linear mixed model fit by REML ['lmerMod']
Formula: Adj ~ Location + (1 | Grp)
   Data: Multilocation
REML criterion at convergence: 9.8205
Random effects:
 Groups   Name        Std.Dev.
 Grp      (Intercept) 0.04067
 Residual             0.22459
Number of obs: 108, groups: Grp, 27
Fixed Effects:
(Intercept)    LocationA    LocationB    LocationC    LocationD
    2.52149      0.47818     -0.22443      0.52712      0.02902
  LocationE    LocationF    LocationG    LocationH
    0.32537      0.73709      0.32098      0.80099
> (fm4Mult <- lmer(Adj ~            Trt + (1|Grp), Multilocation))
Linear mixed model fit by REML ['lmerMod']
Formula: Adj ~ Trt + (1 | Grp)
   Data: Multilocation
REML criterion at convergence: 31.5057
Random effects:
 Groups   Name        Std.Dev.
 Grp      (Intercept) 0.3330
 Residual             0.1916
Number of obs: 108, groups: Grp, 27
Fixed Effects:
(Intercept)         Trt1         Trt2         Trt3
    2.86567      0.05834     -0.18802      0.08379
> (fm5Mult <- lmer(Adj ~ 1              + (1|Grp), Multilocation))
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: Adj ~ 1 + (1 | Grp)
   Data: Multilocation
REML criterion at convergence: 47.3273
Random effects:
 Groups   Name        Std.Dev.
 Grp      (Intercept) 0.3279
 Residual             0.2246
Number of obs: 108, groups: Grp, 27
Fixed Effects:
(Intercept)
      2.854
> anova(fm2Mult)
Analysis of Variance Table
         Df Sum Sq Mean Sq F value
Location  8 7.3768 0.92209  25.115
Trt       3 1.2217 0.40725  11.092
> fm2MultR <- lmer(Adj ~ Trt + (Trt - 1|Location) + (1|Block), Multilocation,
+                  verbose = TRUE)
(NM) 20: f = 61.9198 at    1.00545 0.00545455 0.00545455 0.00545455    1.0054
(NM) 40: f = 59.561 at    1.00545 0.00261822  0.0403054  0.0344762    1.00004
(NM) 60: f = 54.0031 at    1.02396 0.00929341   0.138615  0.0780559   0.98756
(NM) 80: f = 39.4344 at    1.0713 0.0373298  0.307821   0.20942   1.00342  0.
(NM) 100: f = 32.0819 at   1.19531 0.0751749  0.728702  0.453329  0.994316  0
(NM) 120: f = 26.5195 at  1.25611 0.366584 0.603141 0.407144  1.05837 0.87578
(NM) 140: f = 24.4381 at   1.3621 0.665294 0.738454 0.547998   1.1212  1.3044
(NM) 160: f = 24.4381 at   1.3621 0.665294 0.738454 0.547998   1.1212  1.3044
(NM) 180: f = 24.2193 at   1.3499  0.65325   0.7649 0.517308  1.10627  1.1453
(NM) 200: f = 24.0942 at  1.40234 0.763815 0.855302 0.571687  1.11842  1.2674
(NM) 220: f = 22.6873 at  1.45124 0.675951  1.03427  0.50153  1.03942 0.84732
(NM) 240: f = 20.7867 at  1.66872 0.913098  1.33985  0.45005  1.02703 0.56048
(NM) 260: f = 20.689 at  1.82264  1.01623  1.60798 0.378413 0.979706  0.13553
(NM) 280: f = 20.1255 at  1.80393  1.03898  1.49164 0.454599  1.02789 0.58044
(NM) 300: f = 18.352 at  1.83323  1.03575  1.48799 0.419642  1.02885 0.669866
(NM) 320: f = 16.2411 at   1.79771  0.847523   1.50297  0.306929  0.979897  0
(NM) 340: f = 14.2286 at   1.94231   1.04428   1.73304  0.366229  0.995187  0
(NM) 360: f = 14.0233 at    2.0624   1.07514   1.86884  0.342522  0.955776  0
(NM) 380: f = 13.9003 at    2.0849   1.18424   1.98312   0.34784   0.95409  0
(NM) 400: f = 13.676 at   2.18803   1.23441   2.11555  0.297887  0.930266  0.
(NM) 420: f = 13.6232 at   2.04961   1.10473   1.89215  0.305718  0.955703  0
```

```
(NM) 440: f = 13.5372 at  2.12781  1.19533  1.99379 0.296788 0.951442 0.68556
(NM) 460: f = 13.4904 at   2.20434   1.27342   2.05497  0.275888  0.950807  0
(NM) 480: f = 13.4707 at    2.2032   1.26208   2.03241  0.262177  0.953144  0
(NM) 500: f = 13.4025 at   2.26975   1.27669   2.10891  0.239464  0.935694
(NM) 520: f = 13.2212 at   2.36945   1.25332   2.13275  0.207875  0.916464  0
(NM) 540: f = 13.1398 at   2.43055   1.23094   2.08102  0.200132  0.915996  0
(NM) 560: f = 13.1364 at   2.43347   1.21358   2.06449  0.194451   0.91487  0
(NM) 580: f = 13.1154 at   2.39312   1.19974    2.0126  0.186435  0.926064  0
(NM) 600: f = 13.1054 at   2.42565   1.22996   2.06537  0.190615  0.920009  0
(NM) 620: f = 13.0991 at   2.41147   1.20897   2.01764  0.189463  0.926611
(NM) 640: f = 13.0922 at   2.42964   1.22439   2.02352  0.179988  0.928005  0
(NM) 660: f = 13.0639 at   2.43315   1.23191   2.01856  0.173076   0.93278  0
(NM) 680: f = 13.016 at    2.3713   1.22969   2.00382  0.188877  0.946905  0.
(NM) 700: f = 12.9083 at   2.29075   1.13328   1.86767  0.177206  0.978849
(NM) 720: f = 12.8075 at   2.21898   1.09171   1.89915  0.169841  0.984197  0
(NM) 740: f = 12.7992 at   2.19757   1.10341   1.86916  0.178646  0.995988  0
(NM) 760: f = 12.7897 at   2.17796   1.13273   1.90752  0.179228    1.0011  0
(NM) 780: f = 12.7809 at   2.15056    1.1046   1.84015  0.174026    1.0153  0
(NM) 800: f = 12.7804 at   2.12732   1.09436   1.82856  0.183473   1.01688  0
(NM) 820: f = 12.7787 at   2.13662   1.08847   1.83066  0.177947   1.01481  0
(NM) 840: f = 12.7773 at   2.15028   1.10128   1.84278  0.176254   1.01445  0
(NM) 860: f = 12.7725 at   2.15922   1.10712   1.85667  0.177219   1.01065  0
(NM) 880: f = 12.7655 at   2.14918   1.11286   1.86305  0.180553   1.01155  0
(NM) 900: f = 12.7537 at  2.18932  1.12635  1.88674 0.172552  1.00228 0.54552
(NM) 920: f = 12.7291 at  2.17076  1.16096  1.86147 0.185085  1.00762 0.59752
(NM) 940: f = 12.7026 at   2.16097   1.13991   1.83463  0.175591   1.01189  0
(NM) 960: f = 12.6284 at   2.10113   1.17069   1.80221  0.198297   1.01406  0
(NM) 980: f = 12.5335 at   2.03733   1.16124    1.7379  0.207672   1.02195  0
(NM) 1000: f = 12.384 at   2.11367   1.13749   1.78552  0.197318  0.966393  0
(NM) 1020: f = 12.233 at   2.08328    1.2079   1.73936  0.240806  0.936401  0
(NM) 1040: f = 12.173 at   2.11078   1.22627   1.74331  0.261277  0.893935  0
(NM) 1060: f = 12.1041 at   2.07633   1.32742   1.77528  0.279233  0.897747
(NM) 1080: f = 12.035 at    2.1325    1.3692   1.80872  0.291651  0.833455  0
(NM) 1100: f = 12.0201 at   2.15982   1.41312   1.79734  0.287904  0.829344
(NM) 1120: f = 12.0057 at   2.14337   1.40507   1.78759   0.27548  0.839779
(NM) 1140: f = 12.0009 at   2.13059   1.39954   1.76789  0.291957  0.824309
(NM) 1160: f = 11.9937 at   2.12354   1.41228   1.75392  0.287247   0.82142
(NM) 1180: f = 11.9881 at  2.08833  1.39056  1.74483 0.293739   0.8131 0.5975
(NM) 1200: f = 11.9855 at   2.08809   1.38455   1.74466  0.288707  0.820793
(NM) 1220: f = 11.985 at   2.08004   1.37932    1.7397  0.287017  0.821874  0
```

```
(NM) 1240: f = 11.9837 at  2.08386  1.40108  1.74754 0.289483 0.816459   0.58
(NM) 1260: f = 11.9837 at  2.08386  1.40108  1.74754 0.289483 0.816459   0.58
(NM) 1280: f = 11.9837 at   2.08996   1.39612   1.74955  0.288525  0.818697
(NM) 1300: f = 11.9836 at   2.08841   1.39984   1.75106  0.289973  0.817218
(NM) 1320: f = 11.9835 at   2.08973   1.40261   1.75006  0.289534  0.815948
(NM) 1340: f = 11.9835 at   2.08891   1.40083   1.74946  0.289551  0.816277
(NM) 1360: f = 11.9834 at   2.09111   1.40111   1.74937  0.289615  0.815641
(NM) 1380: f = 11.9832 at   2.08996       1.4   1.74752  0.288629  0.815924
(NM) 1400: f = 11.9822 at   2.08829   1.39455   1.74649  0.287041  0.816721
(NM) 1420: f = 11.9812 at   2.09074    1.3991   1.75231   0.28809  0.814558
(NM) 1440: f = 11.9789 at   2.10291   1.39203   1.76035  0.284577  0.813817
(NM) 1460: f = 11.9784 at   2.10452   1.38288   1.76606   0.28424  0.816039
(NM) 1480: f = 11.9734 at   2.13221   1.41877   1.77934  0.285026  0.804117
(NM) 1500: f = 11.9698 at   2.13289   1.42208   1.79342  0.285478  0.802824
(NM) 1520: f = 11.9693 at   2.13183   1.42029   1.78343  0.284999  0.801275
(NM) 1540: f = 11.9664 at  2.14521  1.42269  1.79235 0.284746 0.796568 0.5550
(NM) 1560: f = 11.963 at    2.1601   1.41335   1.79853   0.28385  0.796978  0
(NM) 1580: f = 11.9542 at   2.17395    1.4072   1.79487  0.283013  0.801184
(NM) 1600: f = 11.9287 at   2.18982   1.40132   1.80705  0.289973  0.807197
(NM) 1620: f = 11.9026 at   2.22527   1.42112   1.83807  0.306969  0.808488
(NM) 1640: f = 11.8661 at   2.23857   1.41239   1.84436  0.319065  0.812582
(NM) 1660: f = 11.7914 at   2.18134   1.37563   1.78018  0.322485  0.827421
(NM) 1680: f = 11.6448 at   2.19173   1.30767   1.71513  0.380578  0.846455
(NM) 1700: f = 11.4468 at   2.10631   1.33488   1.64624  0.491925  0.885251
(NM) 1720: f = 11.0259 at  2.06191  1.39526  1.62123 0.543078 0.872248 0.8321
(NM) 1740: f = 10.0475 at    2.10941    1.35954    1.55582   0.778509   0.938
(NM) 1760: f = 9.27268 at  1.95803  1.27274  1.31405  1.07539  1.04039  1.294
(NM) 1780: f = 8.91509 at   2.03355   1.24606   1.42214   1.17822   1.08941
(NM) 1800: f = 8.29709 at  2.03352  1.15665  1.38263  1.12801  1.04244  1.110
(NM) 1820: f = 8.11779 at  1.95655  1.18711  1.38354  1.08507  1.01596  0.984
(NM) 1840: f = 8.0726 at  1.96253  1.17907  1.41558  1.07713  1.02363 0.95632
(NM) 1860: f = 7.96664 at  1.99183  1.22938  1.52611  1.02639 0.996848 0.9385
(NM) 1880: f = 7.87922 at  1.92371  1.18389  1.46657   1.0442  1.01993 0.9363
(NM) 1900: f = 7.85737 at  1.91944  1.19355  1.48516   1.0842  1.03747 0.9910
(NM) 1920: f = 7.83673 at  1.89171  1.18225  1.47285  1.07146  1.04115 0.9860
(NM) 1940: f = 7.79845 at   1.8534  1.13028  1.43044   1.0534  1.05866 0.9759
(NM) 1960: f = 7.78101 at  1.82894  1.09835  1.38622  1.02775  1.06669  0.961
(NM) 1980: f = 7.75116 at  1.83584   1.1105  1.41578  1.02365  1.06507 0.9481
(NM) 2000: f = 7.69582 at   1.8218  1.06492  1.37267 0.982198   1.0768 0.9252
(NM) 2020: f = 7.66571 at  1.82851  1.04047  1.35675  1.01455  1.09577 0.9551
```

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(NM) 2040: f = 7.54979 at  1.89459   1.0323  1.36912  1.01424   1.1004 0.9405
(NM) 2060: f = 7.4556 at   1.88425   1.03975   1.37355   0.99562    1.1156  0
(NM) 2080: f = 7.27001 at   1.90729   1.08678   1.45478    1.0164   1.14602
(NM) 2100: f = 7.14887 at   1.87107   1.12161   1.46779   1.00976   1.14649
(NM) 2120: f = 7.06088 at  1.82261   1.1938  1.44906  1.09486  1.17023 0.9714
(NM) 2140: f = 6.8876 at   1.85403      1.22   1.46988   1.10555   1.18647  0
(NM) 2160: f = 6.8087 at    1.9333   1.26647   1.53804   1.17783   1.17679  0
(NM) 2180: f = 6.64797 at   1.94072   1.32191   1.53544   1.20588   1.20712
(NM) 2200: f = 6.38235 at   1.89789    1.2275   1.50345   1.12161   1.17124
(NM) 2220: f = 5.28253 at  1.93002  1.26748  1.46829  1.25486 0.822962 0.6494
(NM) 2240: f = 3.91833 at    1.786  1.39324  1.45804  1.30288 0.496145 0.4527
(NM) 2260: f = 3.67151 at  1.69175  1.47508  1.47088  1.41085 0.264411 0.4120
(NM) 2280: f = 3.47942 at  1.78106  1.39349  1.46496  1.20416  0.55617 0.4868
(NM) 2300: f = 3.07981 at  1.76468  1.45347  1.53864  1.24433 0.386478 0.2988
(NM) 2320: f = 2.90588 at   1.80163   1.60534   1.58663   1.45143  0.316265
(NM) 2340: f = 2.46793 at   1.76705   1.56042    1.6135   1.35563  0.278209
(NM) 2360: f = 2.44314 at  1.72804  1.54535  1.63088  1.33512 0.212492 0.2729
(NM) 2380: f = 2.44314 at  1.72804  1.54535  1.63088  1.33512 0.212492 0.2729
(NM) 2400: f = 2.39768 at   1.75619   1.51737    1.6167   1.28883  0.308789
(NM) 2420: f = 2.38472 at   1.76387   1.49057   1.60428   1.30066   0.27666
(NM) 2440: f = 2.36176 at    1.75308    1.50101    1.61453    1.30375   0.288
(NM) 2460: f = 2.34672 at   1.74961   1.51342   1.60297   1.31362  0.268366
(NM) 2480: f = 2.29346 at   1.75058   1.50786   1.60221    1.3135   0.31407
(NM) 2500: f = 2.28385 at     1.74011     1.48772     1.58804     1.31791
(NM) 2520: f = 2.25515 at      1.733    1.49054    1.60464    1.33439   0.318
(NM) 2540: f = 2.21317 at    1.72278      1.472    1.60045    1.36301   0.359
(NM) 2560: f = 2.16328 at    1.71638    1.49166    1.63418    1.35117   0.367
(NM) 2580: f = 2.08334 at    1.68688    1.44607    1.62126    1.35717   0.298
(NM) 2600: f = 2.01409 at    1.65951    1.43489    1.62315    1.35938   0.305
(NM) 2620: f = 1.85329 at    1.67381    1.48828    1.62979    1.36232    0.25
(NM) 2640: f = 1.68403 at   1.67739    1.5014    1.6188   1.42776   0.26925
(NM) 2660: f = 1.57893 at   1.72673   1.54207   1.65795   1.47801  0.239536
(NM) 2680: f = 1.45909 at    1.80751    1.59036    1.68377     1.5488   0.223
(NM) 2700: f = 1.45909 at    1.80751    1.59036    1.68377     1.5488   0.223
(NM) 2720: f = 1.45524 at    1.84096    1.60635      1.692    1.56536   0.232
(NM) 2740: f = 1.43092 at    1.84902    1.60903    1.71071    1.55575   0.242
(NM) 2760: f = 1.42293 at     1.8751    1.62999    1.73329    1.58505   0.243
(NM) 2780: f = 1.41627 at    1.89083    1.64553    1.76646    1.59434   0.254
(NM) 2800: f = 1.41378 at     1.8863    1.65314     1.7692    1.61093   0.254
(NM) 2820: f = 1.41179 at    1.88883    1.65235     1.7613    1.60597   0.253
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(NM) 2840: f = 1.41021 at    1.89963    1.65661    1.76696    1.60809   0.251
(NM) 2860: f = 1.409 at    1.90136    1.66553    1.77076    1.61045   0.24458
(NM) 2880: f = 1.40878 at     1.9069    1.67131    1.77717    1.61419   0.248
(NM) 2900: f = 1.40842 at     1.902   1.67066   1.77808   1.61408  0.243593
(NM) 2920: f = 1.4083 at    1.90099    1.66836    1.77466    1.61526   0.2445
(NM) 2940: f = 1.40828 at    1.90092    1.66878    1.77711    1.61481   0.243
(NM) 2960: f = 1.40825 at    1.90121    1.66846    1.77586    1.61502   0.244
(NM) 2980: f = 1.40824 at    1.90081    1.66759    1.77577    1.61388   0.244
(NM) 3000: f = 1.40823 at    1.90101    1.66889    1.77659    1.61519   0.244
(NM) 3020: f = 1.40823 at   1.90119   1.66822   1.77666   1.61482  0.244798
(NM) 3040: f = 1.40822 at    1.90094    1.66844     1.7763    1.61458   0.244
(NM) 3060: f = 1.40822 at    1.90115    1.66893     1.7766    1.61523   0.244
(NM) 3080: f = 1.40821 at     1.9008    1.66867    1.77643    1.61494   0.244
(NM) 3100: f = 1.4082 at    1.90176    1.66958     1.7773    1.61504   0.2443
(NM) 3120: f = 1.4082 at     1.9021    1.66982    1.77743    1.61538   0.2444
(NM) 3140: f = 1.4082 at     1.9018    1.66955    1.77703    1.61523   0.2444
(NM) 3160: f = 1.40819 at    1.90181    1.66938    1.77671    1.61458   0.244
(NM) 3180: f = 1.40817 at    1.90154    1.66914    1.77648    1.61453   0.244
(NM) 3200: f = 1.40814 at    1.90032    1.66857     1.7752    1.61483   0.244
(NM) 3220: f = 1.40809 at    1.90062    1.67033    1.77586    1.61587   0.243
(NM) 3240: f = 1.40807 at    1.89957    1.66932    1.77425    1.61517   0.243
(NM) 3260: f = 1.40805 at    1.90052    1.66912    1.77525    1.61539   0.243
(NM) 3280: f = 1.40801 at    1.90091    1.66965    1.77572    1.61659   0.242
(NM) 3300: f = 1.40796 at    1.89989    1.66908    1.77465    1.61649   0.242
(NM) 3320: f = 1.40792 at    1.89971    1.66804    1.77371    1.61497   0.243
(NM) 3340: f = 1.40784 at    1.90049    1.66807    1.77337    1.61482   0.242
(NM) 3360: f = 1.40778 at     1.8972    1.66547    1.77056    1.61254   0.241
(NM) 3380: f = 1.40767 at   1.89954   1.66661   1.77186   1.61357  0.239924
(NM) 3400: f = 1.40753 at    1.90223    1.66933    1.77562    1.61564   0.240
(NM) 3420: f = 1.40746 at    1.90183    1.66957    1.77647    1.61605   0.239
(NM) 3440: f = 1.40741 at    1.89991    1.66778    1.77392    1.61371   0.240
(NM) 3460: f = 1.40738 at    1.8993   1.66861   1.77343   1.61425  0.242165
(NM) 3480: f = 1.40736 at    1.89931    1.66829    1.77409    1.61438   0.241
(NM) 3500: f = 1.40733 at    1.89951    1.66868    1.77397    1.61521   0.242
(NM) 3520: f = 1.40731 at    1.89942    1.66843    1.77309    1.61477   0.243
(NM) 3540: f = 1.4073 at    1.89932    1.66881    1.77392    1.61556   0.2430
(NM) 3560: f = 1.40729 at    1.89888    1.66851    1.77348    1.61514   0.243
(NM) 3580: f = 1.40726 at    1.89832    1.66749    1.77253    1.61423   0.243
(NM) 3600: f = 1.40724 at    1.89834    1.66711    1.77324    1.61424   0.243
(NM) 3620: f = 1.40722 at    1.89699    1.66568    1.77168    1.61269   0.244
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(NM) 3640: f = 1.4072 at    1.89649    1.66565    1.77197    1.61277   0.2451
(NM) 3660: f = 1.4072 at    1.89656     1.6649    1.77179     1.6122   0.2450
(NM) 3680: f = 1.40719 at    1.89713    1.66557     1.7727     1.6127   0.245
(NM) 3700: f = 1.40719 at    1.89733     1.6659    1.77268    1.61292   0.245
(NM) 3720: f = 1.40718 at     1.89751     1.66595     1.77286     1.61253
(NM) 3740: f = 1.40718 at    1.89749    1.66565    1.77277    1.61208   0.245
(NM) 3760: f = 1.40718 at    1.89772    1.66592    1.77289    1.61235   0.245
(NM) 3780: f = 1.40718 at    1.89765    1.66583    1.77309    1.61239   0.245
(NM) 3800: f = 1.40717 at    1.89761    1.66569    1.77298    1.61252    0.24
(NM) 3820: f = 1.40717 at    1.89769    1.66567    1.77308    1.61245   0.245
(NM) 3840: f = 1.40717 at    1.89766     1.6657    1.77305    1.61247   0.245
(NM) 3860: f = 1.40717 at     1.8978    1.66587    1.77317    1.61248   0.245
(NM) 3880: f = 1.40717 at    1.89776    1.66588    1.77319     1.6126   0.245
(NM) 3900: f = 1.40717 at     1.8977    1.66589    1.77311    1.61264   0.245
(NM) 3920: f = 1.40717 at    1.89786    1.66594    1.77321    1.61262   0.245
(NM) 3940: f = 1.40717 at    1.89777    1.66586    1.77315    1.61256   0.245
(NM) 3960: f = 1.40717 at     1.89757     1.66571     1.77304      1.6125
(NM) 3980: f = 1.40717 at     1.89738     1.66553     1.77297     1.61223
(NM) 4000: f = 1.40717 at    1.89739    1.66557    1.77291    1.61225   0.245
(NM) 4020: f = 1.40717 at    1.89723    1.66537    1.77279    1.61205    0.24
(NM) 4040: f = 1.40717 at     1.89723     1.66544     1.77279     1.61206
(NM) 4060: f = 1.40717 at     1.89714     1.66541     1.77272     1.61191
(NM) 4080: f = 1.40717 at    1.89699    1.66536    1.77256    1.61183   0.245
(NM) 4100: f = 1.40717 at     1.89685     1.66529     1.77237     1.61172
(NM) 4120: f = 1.40717 at     1.89688     1.66525     1.77242     1.61175
(NM) 4140: f = 1.40717 at     1.89693     1.66522     1.77247     1.61167
(NM) 4160: f = 1.40717 at     1.89704     1.66532     1.77252     1.61176
(NM) 4180: f = 1.40717 at     1.89698      1.6653      1.7725     1.61172
(NM) 4200: f = 1.40717 at     1.89701     1.66532     1.77251     1.61177
(NM) 4220: f = 1.40717 at     1.89698     1.66529      1.7725     1.61175
(NM) 4240: f = 1.40717 at      1.897    1.66532    1.77252    1.61178   0.245
(NM) 4260: f = 1.40717 at       1.897     1.66531     1.77252     1.61179
(NM) 4280: f = 1.40717 at     1.89699     1.66531     1.77252     1.61179
(NM) 4300: f = 1.40717 at       1.897     1.66531     1.77251     1.61177
(NM) 4320: f = 1.40717 at       1.897     1.66531     1.77251     1.61177
(NM) 4340: f = 1.40717 at       1.897     1.66532     1.77252     1.61179
(NM) 4360: f = 1.40717 at     1.89701     1.66532     1.77252     1.61178
(NM) 4380: f = 1.40717 at     1.89701     1.66532     1.77252     1.61178
(NM) 4400: f = 1.40717 at     1.89702     1.66532     1.77252     1.61178
(NM) 4420: f = 1.40717 at     1.89702     1.66532     1.77252     1.61179
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(NM) 4440: f = 1.40717 at     1.89703     1.66532     1.77252     1.61179
(NM) 4460: f = 1.40717 at     1.89705     1.66534     1.77254      1.6118
(NM) 4480: f = 1.40717 at     1.89705     1.66534     1.77254     1.61181
(NM) 4500: f = 1.40717 at     1.89704     1.66534     1.77254      1.6118
(NM) 4520: f = 1.40717 at     1.89703     1.66533     1.77253     1.61179
(NM) 4540: f = 1.40717 at     1.89704     1.66534     1.77254      1.6118
(NM) 4560: f = 1.40717 at     1.89702     1.66532     1.77252      1.6118
(NM) 4580: f = 1.40717 at     1.89703     1.66532     1.77252      1.6118
(NM) 4600: f = 1.40717 at    1.89701    1.66529     1.7725    1.61177   0.245
(NM) 4620: f = 1.40717 at       1.897     1.66529      1.7725     1.61177
(NM) 4640: f = 1.40717 at       1.897     1.66529     1.77249     1.61177
(NM) 4660: f = 1.40717 at     1.89697     1.66527     1.77245     1.61175
(NM) 4680: f = 1.40717 at     1.89697     1.66527     1.77245     1.61175
(NM) 4700: f = 1.40717 at     1.89697     1.66527     1.77245     1.61174
(NM) 4720: f = 1.40717 at     1.89697     1.66527     1.77245     1.61174
(NM) 4740: f = 1.40717 at     1.89697     1.66526     1.77245     1.61173
(NM) 4760: f = 1.40717 at     1.89698     1.66527     1.77245     1.61174
(NM) 4780: f = 1.40717 at     1.89698     1.66526     1.77245     1.61174
(NM) 4800: f = 1.40717 at     1.89699     1.66527     1.77246     1.61174
(NM) 4820: f = 1.40717 at     1.89698     1.66527     1.77246     1.61175
(NM) 4840: f = 1.40717 at     1.89699     1.66527     1.77246     1.61175
(NM) 4860: f = 1.40717 at     1.89698     1.66527     1.77245     1.61175
(NM) 4880: f = 1.40717 at     1.89698     1.66527     1.77245     1.61175
(NM) 4900: f = 1.40717 at     1.89698     1.66527     1.77246     1.61175
(NM) 4920: f = 1.40717 at     1.89698     1.66527     1.77245     1.61175
(NM) 4940: f = 1.40717 at     1.89698     1.66527     1.77246     1.61175
(NM) 4960: f = 1.40717 at     1.89698     1.66527     1.77245     1.61175
(NM) 4980: f = 1.40717 at     1.89698     1.66527     1.77245     1.61175
(NM) 5000: f = 1.40717 at     1.89698     1.66528     1.77245     1.61175
(NM) 5020: f = 1.40717 at     1.89698     1.66527     1.77245     1.61176
(NM) 5040: f = 1.40717 at     1.89698     1.66527     1.77245     1.61175
(NM) 5060: f = 1.40717 at     1.89698     1.66527     1.77245     1.61175
(NM) 5080: f = 1.40717 at     1.89698     1.66526     1.77244     1.61174
(NM) 5100: f = 1.40717 at     1.89698     1.66527     1.77244     1.61175
(NM) 5120: f = 1.40717 at     1.89699     1.66527     1.77244     1.61175
(NM) 5140: f = 1.40717 at       1.897     1.66528     1.77245     1.61175
(NM) 5160: f = 1.40717 at       1.897     1.66529     1.77245     1.61177
(NM) 5180: f = 1.40717 at     1.89701     1.66529     1.77245     1.61177
(NM) 5200: f = 1.40717 at     1.89701     1.66529     1.77245     1.61177
(NM) 5220: f = 1.40717 at       1.897     1.66529     1.77245     1.61176
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(NM) 5240: f = 1.40717 at     1.89701     1.66529     1.77246     1.61176
(NM) 5260: f = 1.40717 at     1.89701     1.66529     1.77246     1.61177
(NM) 5280: f = 1.40717 at     1.89701     1.66529     1.77246     1.61177
(NM) 5300: f = 1.40717 at     1.89703      1.6653     1.77247     1.61177
(NM) 5320: f = 1.40717 at     1.89703     1.66531     1.77248     1.61177
(NM) 5340: f = 1.40717 at     1.89703     1.66531     1.77247     1.61177
(NM) 5360: f = 1.40717 at     1.89704     1.66531     1.77249     1.61179
(NM) 5380: f = 1.40717 at     1.89703     1.66531     1.77248     1.61178
(NM) 5400: f = 1.40717 at    1.89703     1.6653    1.77247    1.61177   0.245
(NM) 5420: f = 1.40717 at     1.89703     1.66531     1.77248     1.61178
(NM) 5440: f = 1.40717 at     1.89701      1.6653     1.77247     1.61176
(NM) 5460: f = 1.40717 at     1.89702     1.66532     1.77247     1.61177
(NM) 5480: f = 1.40717 at     1.89702     1.66531     1.77246     1.61177
(NM) 5500: f = 1.40717 at       1.897     1.66531     1.77246     1.61176
(NM) 5520: f = 1.40717 at     1.89701     1.66532     1.77246     1.61178
(NM) 5540: f = 1.40717 at     1.89701     1.66531     1.77245     1.61178
(NM) 5560: f = 1.40717 at       1.897     1.66531     1.77246     1.61178
(NM) 5580: f = 1.40717 at     1.89702     1.66533     1.77251     1.61181
(NM) 5600: f = 1.40717 at     1.89704     1.66533     1.77251     1.61181
(NM) 5620: f = 1.40717 at    1.89705    1.66534    1.77252    1.61182   0.245
(NM) 5640: f = 1.40717 at     1.89705     1.66534     1.77251     1.61184
(NM) 5660: f = 1.40717 at     1.89706     1.66534     1.77251     1.61183
(NM) 5680: f = 1.40717 at     1.89706     1.66534     1.77252     1.61185
(NM) 5700: f = 1.40717 at     1.89706     1.66534     1.77252     1.61184
(NM) 5720: f = 1.40717 at     1.89705     1.66534     1.77252     1.61184
(NM) 5740: f = 1.40717 at     1.89705     1.66533     1.77251     1.61183
(NM) 5760: f = 1.40717 at     1.89705     1.66534     1.77252     1.61184
(NM) 5780: f = 1.40717 at     1.89705     1.66534     1.77252     1.61184
(NM) 5800: f = 1.40717 at     1.89705     1.66534     1.77252     1.61184
(NM) 5820: f = 1.40717 at     1.89705     1.66534     1.77252     1.61184
(NM) 5840: f = 1.40717 at     1.89705     1.66534     1.77252     1.61184
(NM) 5860: f = 1.40717 at     1.89705     1.66534     1.77252     1.61183
(NM) 5880: f = 1.40717 at    1.89705    1.66534    1.77252    1.61184   0.245
(NM) 5900: f = 1.40717 at     1.89701     1.66531     1.77249     1.61181
(NM) 5920: f = 1.40717 at     1.89704     1.66534     1.77251     1.61184
(NM) 5940: f = 1.40717 at     1.89704     1.66534     1.77254     1.61186
(NM) 5960: f = 1.40717 at     1.89706     1.66536     1.77257     1.61189
(NM) 5980: f = 1.40717 at     1.89702     1.66532     1.77257     1.61188
(NM) 6000: f = 1.40717 at       1.897     1.66529     1.77255     1.61183
(NM) 6020: f = 1.40717 at     1.89705     1.66535      1.7726      1.6118
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(NM) 6040: f = 1.40717 at     1.89709     1.66539     1.77266     1.61181
(NM) 6060: f = 1.40717 at      1.8971     1.66539     1.77266     1.61179
(NM) 6080: f = 1.40717 at     1.89706     1.66536     1.77261     1.61173
(NM) 6100: f = 1.40717 at     1.89712     1.66539     1.77263     1.61177
(NM) 6120: f = 1.40717 at     1.89707     1.66535     1.77261     1.61178
(NM) 6140: f = 1.40717 at     1.89713     1.66538     1.77263      1.6118
(NM) 6160: f = 1.40717 at     1.89722     1.66545     1.77273     1.61184
(NM) 6180: f = 1.40717 at     1.89718     1.66538     1.77269     1.61177
(NM) 6200: f = 1.40717 at     1.89717     1.66536     1.77267     1.61175
(NM) 6220: f = 1.40717 at     1.89715     1.66536     1.77267     1.61174
(NM) 6240: f = 1.40717 at    1.89711    1.66531    1.77265    1.61169   0.245
(NM) 6260: f = 1.40717 at     1.89713     1.66532     1.77265     1.61169
(NM) 6280: f = 1.40717 at     1.89712     1.66529     1.77266      1.6117
(NM) 6300: f = 1.40717 at     1.89715     1.66532     1.77268     1.61172
(NM) 6320: f = 1.40717 at      1.8972     1.66534     1.77269     1.61174
(NM) 6340: f = 1.40717 at     1.89727     1.66536      1.7727     1.61175
(NM) 6360: f = 1.40717 at     1.89725     1.66532     1.77269     1.61173
(NM) 6380: f = 1.40717 at     1.89724     1.66535     1.77271     1.61175
(NM) 6400: f = 1.40717 at     1.89728     1.66537     1.77274     1.61178
(NM) 6420: f = 1.40717 at     1.89727     1.66537     1.77273     1.61177
(NM) 6440: f = 1.40717 at     1.89727     1.66537     1.77275     1.61176
(NM) 6460: f = 1.40717 at     1.89725     1.66533     1.77273     1.61173
(NM) 6480: f = 1.40717 at     1.89726     1.66534     1.77275     1.61174
(NM) 6500: f = 1.40717 at     1.89724      1.6653     1.77272      1.6117
(NM) 6520: f = 1.40717 at    1.89722    1.66527    1.77271     1.6117   0.245
(NM) 6540: f = 1.40717 at    1.89716    1.66522    1.77267    1.61167   0.245
(NM) 6560: f = 1.40717 at    1.89716    1.66523    1.77264    1.61168   0.245
(NM) 6580: f = 1.40717 at    1.89714    1.66519    1.77262     1.6117   0.245
(NM) 6600: f = 1.40717 at    1.89716    1.66521    1.77263    1.61171   0.245
(NM) 6620: f = 1.40717 at    1.89714     1.6652    1.77261     1.6117   0.245
(NM) 6640: f = 1.40717 at     1.8972    1.66525    1.77265    1.61174   0.245
(NM) 6660: f = 1.40717 at    1.89719    1.66524    1.77264    1.61174   0.245
(NM) 6680: f = 1.40717 at    1.89722    1.66527    1.77267    1.61177   0.245
(NM) 6700: f = 1.40717 at     1.8972    1.66524    1.77266    1.61176   0.245
(NM) 6720: f = 1.40717 at    1.89722    1.66524    1.77269    1.61176   0.245
(NM) 6740: f = 1.40717 at     1.8973     1.6653    1.77273    1.61182   0.245
(NM) 6760: f = 1.40717 at    1.89731    1.66529    1.77278    1.61178   0.245
(NM) 6780: f = 1.40717 at    1.89732    1.66532    1.77275    1.61181   0.245
(NM) 6800: f = 1.40717 at    1.89724    1.66527    1.77268    1.61173   0.245
(NM) 6820: f = 1.40717 at    1.89728    1.66528    1.77269    1.61174   0.245
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(NM) 6840: f = 1.40717 at    1.89731    1.66522    1.77269    1.61171   0.245
(NM) 6860: f = 1.40717 at    1.89726    1.66519    1.77263    1.61168   0.245
(NM) 6880: f = 1.40717 at    1.89733    1.66523    1.77265    1.61165   0.245
(NM) 6900: f = 1.40717 at    1.89739    1.66521    1.77272    1.61162   0.245
(NM) 6920: f = 1.40717 at    1.89734    1.66511    1.77263    1.61151   0.245
(NM) 6940: f = 1.40717 at    1.89722    1.66508    1.77252    1.61147   0.245
(NM) 6960: f = 1.40717 at     1.89718     1.66503     1.77246     1.61143
(NM) 6980: f = 1.40717 at    1.89724    1.66501     1.7725    1.61145   0.245
(NM) 7000: f = 1.40717 at    1.89728    1.66506    1.77254    1.61151   0.245
(NM) 7020: f = 1.40717 at    1.89724    1.66504    1.77252    1.61149   0.245
(NM) 7040: f = 1.40717 at     1.89715     1.66503     1.77247     1.61147
(NM) 7060: f = 1.40716 at     1.89717     1.66508     1.77257     1.61153
(NM) 7080: f = 1.40716 at     1.8972    1.66511    1.77257    1.61156   0.245
(NM) 7100: f = 1.40716 at     1.89729     1.66522     1.77264     1.61163
(NM) 7120: f = 1.40716 at     1.89735     1.66526     1.77276     1.61173
(NM) 7140: f = 1.40716 at     1.89731     1.66522     1.77273     1.61165
(NM) 7160: f = 1.40716 at     1.89736     1.66523     1.77276     1.61167
(NM) 7180: f = 1.40716 at      1.8974     1.66525     1.77275     1.61164
(NM) 7200: f = 1.40716 at      1.8974     1.66525     1.77276     1.61165
(NM) 7220: f = 1.40716 at     1.89744      1.6653     1.77277     1.61169
(NM) 7240: f = 1.40716 at     1.89747     1.66529     1.77278     1.61168
(NM) 7260: f = 1.40716 at     1.89745     1.66525     1.77273     1.61162
(NM) 7280: f = 1.40716 at     1.89749     1.66528     1.77279      1.6117
(NM) 7300: f = 1.40716 at     1.89754      1.6653     1.77287     1.61172
(NM) 7320: f = 1.40716 at     1.89758     1.66534     1.77292     1.61177
(NM) 7340: f = 1.40716 at    1.89756    1.66539    1.77291    1.61177   0.245
(NM) 7360: f = 1.40716 at    1.89741    1.66532    1.77285    1.61167   0.245
(NM) 7380: f = 1.40716 at      1.8974     1.66536     1.77286     1.61182
(NM) 7400: f = 1.40716 at     1.89723     1.66539     1.77283     1.61175
(NM) 7420: f = 1.40716 at     1.8974    1.66546    1.77288    1.61184    0.24
(NM) 7440: f = 1.40716 at     1.89741     1.66554     1.77297     1.61192
(NM) 7460: f = 1.40716 at     1.89749     1.66565     1.77299     1.61202
(NM) 7480: f = 1.40716 at    1.89752    1.66569    1.77306    1.61206   0.245
(NM) 7500: f = 1.40716 at     1.89756     1.66571     1.77306     1.61204
(NM) 7520: f = 1.40716 at    1.89747    1.66578    1.77298    1.61198   0.246
(NM) 7540: f = 1.40716 at    1.89744    1.66565     1.7729    1.61172   0.246
(NM) 7560: f = 1.40716 at    1.89734    1.66547    1.77271    1.61159   0.246
(NM) 7580: f = 1.40716 at    1.89734    1.66547    1.77271    1.61159   0.246
(NM) 7600: f = 1.40716 at    1.89744    1.66554    1.77284    1.61169   0.246
(NM) 7620: f = 1.40716 at     1.8974    1.66549    1.77274    1.61164   0.246
```

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(NM) 7640: f = 1.40716 at    1.89731    1.66542    1.77268    1.61155   0.246
(NM) 7660: f = 1.40716 at    1.89732    1.66543     1.7727     1.6116   0.246
(NM) 7680: f = 1.40716 at    1.89729    1.66546     1.7727    1.61162   0.246
(NM) 7700: f = 1.40716 at     1.8973    1.66543     1.7727     1.6116   0.246
(NM) 7720: f = 1.40716 at    1.89736    1.66547    1.77272    1.61164   0.246
(NM) 7740: f = 1.40716 at    1.89736    1.66548    1.77273    1.61166   0.246
(NM) 7760: f = 1.40716 at    1.89734     1.6655    1.77273    1.61166   0.246
(NM) 7780: f = 1.40716 at    1.89741    1.66553    1.77275    1.61169   0.246
(NM) 7800: f = 1.40716 at     1.8974    1.66557    1.77274    1.61173   0.246
(NM) 7820: f = 1.40716 at    1.89738    1.66553     1.7727    1.61169   0.246
(NM) 7840: f = 1.40716 at    1.89737    1.66545    1.77265     1.6116   0.246
(NM) 7860: f = 1.40716 at    1.89746    1.66552    1.77269    1.61168   0.246
(NM) 7880: f = 1.40716 at    1.89739    1.66546    1.77265    1.61166   0.246
(NM) 7900: f = 1.40716 at    1.89757    1.66565    1.77281    1.61182   0.246
(NM) 7920: f = 1.40716 at    1.89754     1.6656    1.77279    1.61178    0.24
(NM) 7940: f = 1.40716 at    1.89756    1.66571    1.77285    1.61185    0.24
(NM) 7960: f = 1.40716 at    1.89759    1.66567    1.77291    1.61178   0.246
(NM) 7980: f = 1.40716 at    1.89774    1.66576      1.773    1.61185   0.246
(NM) 8000: f = 1.40716 at    1.89763    1.66569    1.77298     1.6118   0.246
(NM) 8020: f = 1.40716 at    1.89766    1.66574    1.77299    1.61185   0.246
(NM) 8040: f = 1.40716 at    1.89767    1.66572      1.773    1.61185   0.246
(NM) 8060: f = 1.40716 at    1.89768    1.66576    1.77304     1.6119   0.246
(NM) 8080: f = 1.40716 at    1.89767    1.66576    1.77304    1.61191   0.246
(NM) 8100: f = 1.40716 at     1.8977    1.66575    1.77305    1.61198   0.246
(NM) 8120: f = 1.40716 at     1.8977    1.66575    1.77305    1.61198   0.246
(NM) 8140: f = 1.40716 at    1.89769    1.66573    1.77303    1.61198   0.246
(NM) 8160: f = 1.40716 at     1.8977     1.6657    1.77302    1.61195   0.246
(NM) 8180: f = 1.40716 at    1.89765    1.66568      1.773    1.61193   0.246
(NM) 8200: f = 1.40716 at     1.8977    1.66571    1.77303    1.61195   0.246
(NM) 8220: f = 1.40716 at    1.89767    1.66567    1.77301    1.61193   0.246
(NM) 8240: f = 1.40716 at     1.8977     1.6657    1.77304    1.61195   0.246
(NM) 8260: f = 1.40716 at    1.89769    1.66569    1.77303    1.61195   0.246
(NM) 8280: f = 1.40716 at     1.8977     1.6657    1.77303    1.61195   0.246
(NM) 8300: f = 1.40716 at     1.8977     1.6657    1.77304    1.61196   0.246
(NM) 8320: f = 1.40716 at     1.8977    1.66571    1.77304    1.61196   0.246
(NM) 8340: f = 1.40716 at    1.89769     1.6657    1.77303    1.61195   0.246
(NM) 8360: f = 1.40716 at     1.8977    1.66571    1.77305    1.61197    0.24
(NM) 8380: f = 1.40716 at     1.8977     1.6657    1.77304    1.61196   0.246
(NM) 8400: f = 1.40716 at    1.89769    1.66571    1.77303    1.61196   0.246
(NM) 8420: f = 1.40716 at    1.89769     1.6657    1.77304    1.61197   0.246
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(NM) 8440: f = 1.40716 at    1.89769    1.66571    1.77304    1.61197   0.246
(NM) 8460: f = 1.40716 at    1.89772    1.66572    1.77305    1.61199   0.246
(NM) 8480: f = 1.40716 at    1.89769    1.66571    1.77304    1.61198   0.246
(NM) 8500: f = 1.40716 at    1.89768     1.6657    1.77304    1.61198   0.246
(NM) 8520: f = 1.40716 at    1.89769    1.66571    1.77304    1.61199   0.246
(NM) 8540: f = 1.40716 at    1.89769     1.6657    1.77304    1.61198   0.246
(NM) 8560: f = 1.40716 at    1.89768     1.6657    1.77304    1.61198   0.246
(NM) 8580: f = 1.40716 at    1.89768     1.6657    1.77304    1.61198   0.246
(NM) 8600: f = 1.40716 at    1.89767     1.6657    1.77304    1.61197   0.246
(NM) 8620: f = 1.40716 at    1.89768    1.66569    1.77304    1.61196   0.246
(NM) 8640: f = 1.40716 at    1.89768    1.66569    1.77305    1.61196   0.246
(NM) 8660: f = 1.40716 at    1.89768     1.6657    1.77305    1.61197   0.246
(NM) 8680: f = 1.40716 at    1.89766    1.66568    1.77304    1.61196   0.246
(NM) 8700: f = 1.40716 at    1.89767    1.66567    1.77303    1.61196    0.24
(NM) 8720: f = 1.40716 at    1.89765    1.66567    1.77302    1.61197   0.246
(NM) 8740: f = 1.40716 at    1.89765    1.66566    1.77302    1.61196   0.246
(NM) 8760: f = 1.40716 at    1.89765    1.66566    1.77302    1.61196   0.246
(NM) 8780: f = 1.40716 at    1.89764    1.66564    1.77301    1.61194   0.246
(NM) 8800: f = 1.40716 at    1.89762    1.66563    1.77298    1.61193   0.246
(NM) 8820: f = 1.40716 at    1.89759     1.6656    1.77294    1.61189   0.246
(NM) 8840: f = 1.40716 at   1.89759   1.66561   1.77295    1.6119  0.246886
(NM) 8860: f = 1.40716 at    1.89758    1.66558    1.77293    1.61188   0.246
(NM) 8880: f = 1.40716 at    1.89757    1.66558    1.77291    1.61186   0.246
(NM) 8900: f = 1.40716 at    1.89759    1.66558    1.77291    1.61186   0.246
(NM) 8920: f = 1.40716 at    1.89759    1.66559    1.77293    1.61186   0.246
(NM) 8940: f = 1.40716 at    1.89759     1.6656    1.77293    1.61187   0.246
(NM) 8960: f = 1.40716 at     1.8976     1.6656    1.77292    1.61187   0.246
(NM) 8980: f = 1.40716 at    1.89759    1.66559    1.77291    1.61187   0.246
(NM) 9000: f = 1.40716 at   1.89761   1.66561   1.77293   1.61188  0.246901
(NM) 9020: f = 1.40716 at    1.89762    1.66561    1.77292    1.61188   0.246
(NM) 9040: f = 1.40716 at    1.89761    1.66559    1.77289    1.61186   0.246
(NM) 9060: f = 1.40716 at    1.89759    1.66559     1.7729    1.61187   0.246
(NM) 9080: f = 1.40716 at    1.89751    1.66553    1.77283    1.61179   0.246
(NM) 9100: f = 1.40716 at    1.89752    1.66554    1.77286    1.61183   0.246
(NM) 9120: f = 1.40716 at    1.89755    1.66557    1.77286    1.61184   0.247
(NM) 9140: f = 1.40715 at    1.89756    1.66556    1.77289     1.6119   0.246
(NM) 9160: f = 1.40715 at    1.89762    1.66556    1.77287    1.61192   0.247
(NM) 9180: f = 1.40715 at     1.8976    1.66553    1.77286    1.61193   0.247
(NM) 9200: f = 1.40715 at    1.89763    1.66551    1.77285    1.61193   0.247
(NM) 9220: f = 1.40715 at    1.89759    1.66548    1.77279    1.61188   0.246
```

```
(NM) 9240: f = 1.40715 at    1.89754    1.66537    1.77273    1.61185   0.246
(NM) 9260: f = 1.40715 at    1.89756    1.66539    1.77278    1.61188   0.246
(NM) 9280: f = 1.40715 at    1.89763    1.66542    1.77283    1.61193   0.246
(NM) 9300: f = 1.40715 at    1.89771    1.66545    1.77287    1.61195   0.247
(NM) 9320: f = 1.40715 at    1.89776    1.66548    1.77291    1.61197   0.247
(NM) 9340: f = 1.40715 at    1.89776    1.66541    1.77289    1.61193   0.247
(NM) 9360: f = 1.40715 at    1.89773    1.66537    1.77284    1.61189   0.247
(NM) 9380: f = 1.40715 at    1.89772    1.66537    1.77282    1.61187   0.247
(NM) 9400: f = 1.40715 at    1.89768    1.66534     1.7728    1.61186   0.247
(NM) 9420: f = 1.40715 at    1.89759    1.66526     1.7727    1.61179   0.247
(NM) 9440: f = 1.40715 at    1.89766    1.66527    1.77268    1.61183   0.247
(NM) 9460: f = 1.40715 at    1.89766    1.66526    1.77263    1.61187   0.247
(NM) 9480: f = 1.40715 at     1.8977    1.66529    1.77264    1.61191   0.247
(NM) 9500: f = 1.40715 at    1.89776    1.66534    1.77266    1.61199   0.247
(NM) 9520: f = 1.40715 at    1.89769    1.66543    1.77272    1.61209   0.247
(NM) 9540: f = 1.40715 at    1.89776    1.66538    1.77268    1.61206   0.247
(NM) 9560: f = 1.40715 at    1.89779    1.66544    1.77269    1.61207   0.247
(NM) 9580: f = 1.40715 at    1.89783    1.66547     1.7727    1.61206   0.247
(NM) 9600: f = 1.40715 at    1.89777    1.66547    1.77265      1.612   0.247
(NM) 9620: f = 1.40715 at    1.89782    1.66551    1.77269    1.61207   0.247
(NM) 9640: f = 1.40715 at    1.89777    1.66548    1.77262    1.61201   0.247
(NM) 9660: f = 1.40715 at    1.89774    1.66545    1.77262    1.61202   0.247
(NM) 9680: f = 1.40715 at    1.89771     1.6654    1.77259    1.61197   0.247
(NM) 9700: f = 1.40715 at    1.89769    1.66542    1.77261    1.61195    0.24
(NM) 9720: f = 1.40715 at    1.89769    1.66538     1.7726    1.61195   0.247
(NM) 9740: f = 1.40715 at    1.89761    1.66538    1.77262    1.61192   0.247
(NM) 9760: f = 1.40715 at    1.89768    1.66544    1.77265    1.61204   0.247
(NM) 9780: f = 1.40715 at    1.89759    1.66532    1.77266      1.612   0.247
(NM) 9800: f = 1.40715 at    1.89761     1.6654    1.77268    1.61206   0.247
(NM) 9820: f = 1.40715 at     1.89744     1.66538     1.77258     1.61206
(NM) 9840: f = 1.40715 at     1.89719     1.66536     1.77257     1.61205
(NM) 9860: f = 1.40715 at     1.89726     1.66533     1.77241     1.61197
(NM) 9880: f = 1.40715 at     1.89734     1.66533     1.77245     1.61203
(NM) 9900: f = 1.40715 at      1.8972      1.6653     1.77242     1.61202
(NM) 9920: f = 1.40715 at     1.89724     1.66524      1.7724     1.61199
(NM) 9940: f = 1.40715 at     1.89722     1.66522     1.77238       1.612
(NM) 9960: f = 1.40715 at     1.89726     1.66522     1.77241       1.612
(NM) 9980: f = 1.40715 at     1.89724     1.66523      1.7724     1.61198
(NM) 10000: f = 1.40715 at     1.89725     1.66521     1.77239     1.61198
> ## non convergence in 10000 evaluations
```

```
> fm2MultR
Linear mixed model fit by REML ['lmerMod']
Formula: Adj ~ Trt + (Trt - 1 | Location) + (1 | Block)
   Data: Multilocation
REML criterion at convergence: 1.4071
Random effects:
 Groups   Name        Std.Dev. Corr
 Location Trt1        0.3687
          Trt2        0.3271   0.99
          Trt3        0.3451   1.00 1.00
          Trt4        0.3378   0.93 0.97 0.95
 Block    (Intercept) 0.0000
 Residual             0.1943
Number of obs: 108, groups: Location, 9; Block, 3
Fixed Effects:
(Intercept)         Trt1         Trt2         Trt3
    2.86567      0.05834     -0.18802      0.08379
```

I PBIB

```
> str(PBIB)
'data.frame':        60 obs. of  3 variables:
 $ response : num  2.4 2.5 2.6 2 2.7 2.8 2.4 2.7 2.6 2.8 ...
 $ Treatment: Factor w/ 15 levels "1","10","11",..: 7 15 1 5 11 13 14 1 2 1 .
 $ Block    : Factor w/ 15 levels "1","10","11",..: 1 1 1 1 8 8 8 8 9 9 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 response ~ Treatment | Block
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ order.groups: logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      : list()
  ..$ units       : list()
> ## compare with output 1.7  pp. 24-25
> (fm1PBIB <- lmer(response ~ Treatment + (1 | Block), PBIB))
Linear mixed model fit by REML ['lmerMod']
Formula: response ~ Treatment + (1 | Block)
   Data: PBIB
REML criterion at convergence: 51.9849
```

```
Random effects:
 Groups   Name        Std.Dev.
 Block    (Intercept) 0.2157
 Residual             0.2925
Number of obs: 60, groups: Block, 15
Fixed Effects:
(Intercept)   Treatment1  Treatment10  Treatment11  Treatment12
   2.891311    -0.073789    -0.400250     0.007387     0.161510
Treatment13  Treatment14  Treatment15   Treatment2   Treatment3
  -0.273542    -0.400000    -0.032078    -0.485996    -0.436368
 Treatment4   Treatment5   Treatment6   Treatment7   Treatment8
  -0.107482    -0.086413     0.019382    -0.102327    -0.109706
```

J SIMS

```
> str(SIMS)
'data.frame':        3691 obs. of  3 variables:
 $ Pretot: num  29 38 31 31 29 23 23 33 30 32 ...
 $ Gain  : num  2 0 6 6 5 9 7 2 1 3 ...
 $ Class : Factor w/ 190 levels "1","10","100",..: 1 1 1 1 1 1 1 1 1 1 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 Gain ~ Pretot | Class
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ order.groups: logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      :List of 2
  .. ..$ Pretot: chr "Sum of pre-test core item scores"
  .. ..$ Gain  : chr "Gain in mathematics achievement score"
  ..$ units       : list()
> ## compare to output 7.4, p. 262
> (fm1SIMS <- lmer(Gain ~ Pretot + (Pretot | Class), SIMS))
Linear mixed model fit by REML ['lmerMod']
Formula: Gain ~ Pretot + (Pretot | Class)
   Data: SIMS
REML criterion at convergence: 22380.57
Random effects:
 Groups   Name        Std.Dev. Corr
 Class    (Intercept) 3.80651
```

```
          Pretot      0.09593  -0.64
 Residual             4.71548
Number of obs: 3691, groups: Class, 190
Fixed Effects:
(Intercept)       Pretot
      7.060       -0.186
```